Lecture 2 Intensity Transformations

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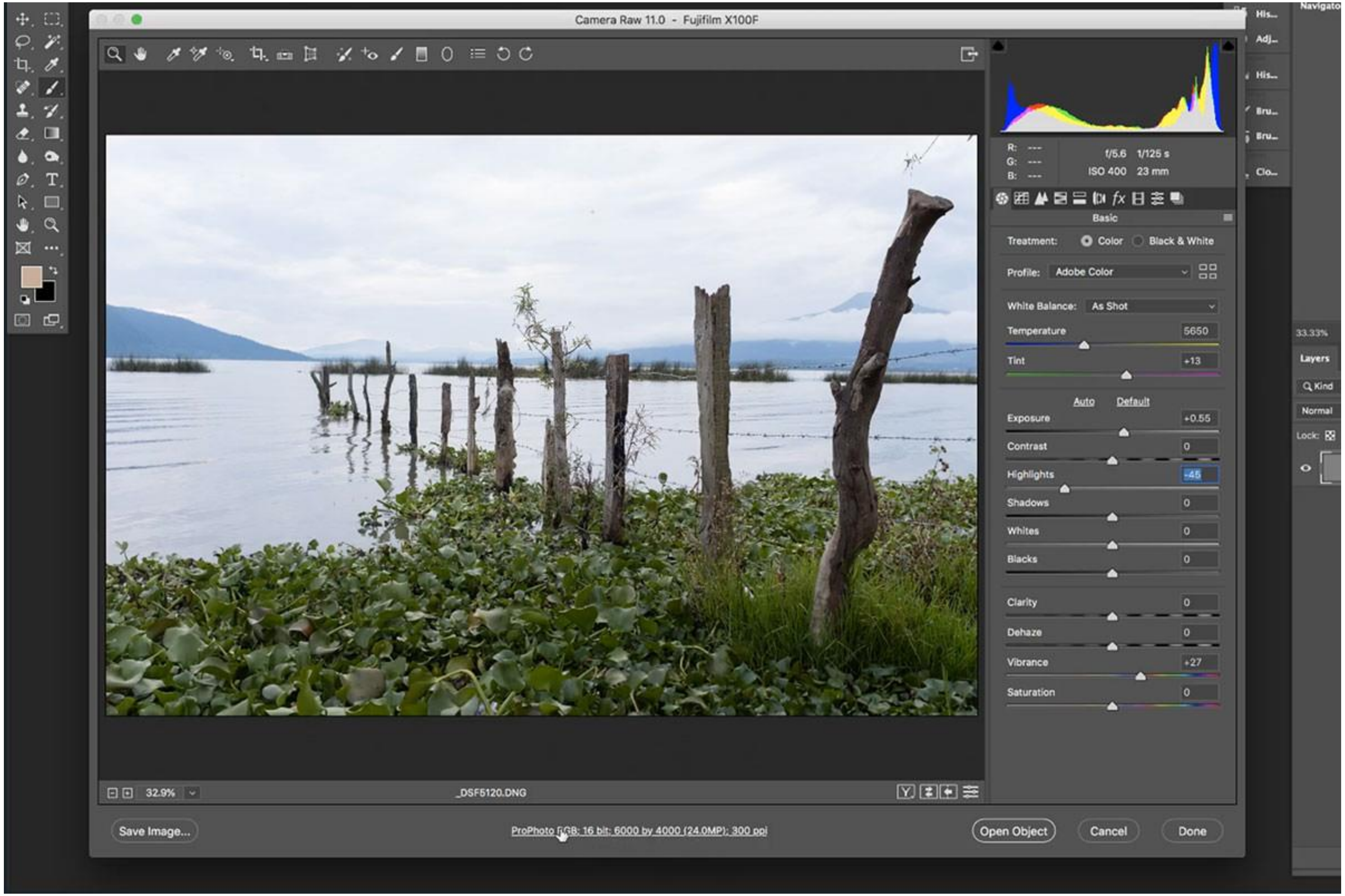
Photoshop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Outline

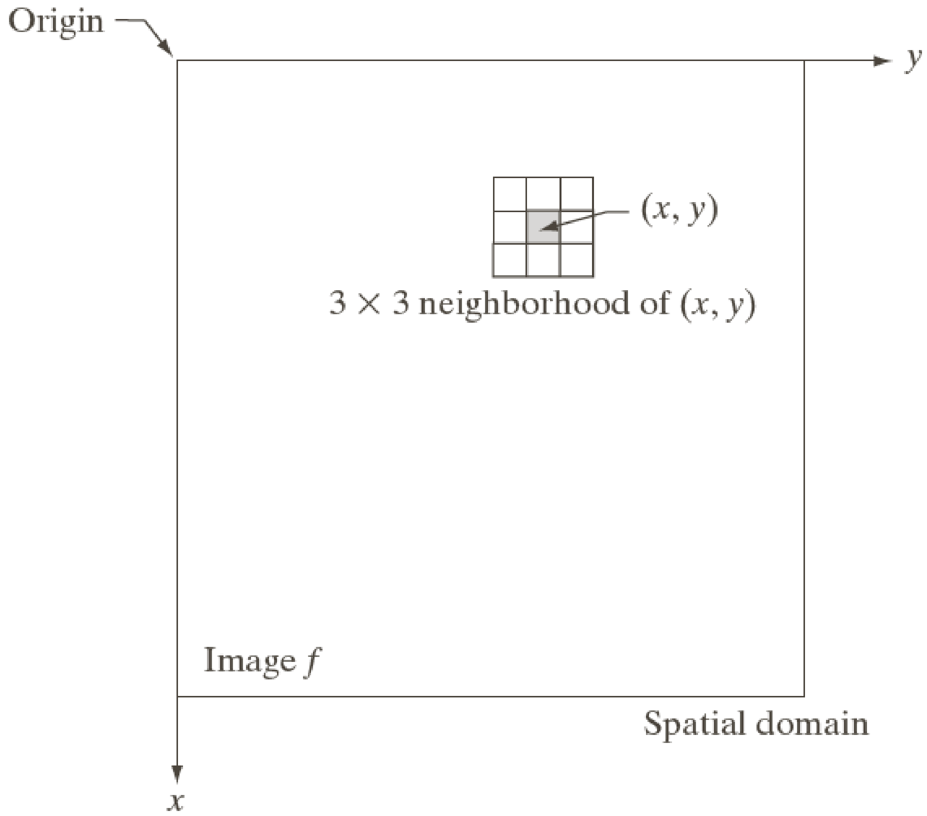
- Background
- Intensity Transformation
  - Image Negatives
  - Log Transformation
  - Power-Law Transformations
  - Piecewise-Linear Transformation
- Histogram Processing
  - Histogram Equalization
  - Histogram Specification

Background

- Spatial domain
  - Operate directly on pixels
  - Contrast to transform domain, e.g. Fourier transform
- General expression

$$g(x, y) = T[f(x, y)]$$

  - $\boldsymbol{f(x, y)}$: input image
  - $\boldsymbol{g(x, y)}$: output image
  - $\boldsymbol{T}[\cdot]$: operator on f, over a neighborhood of point (x, y)



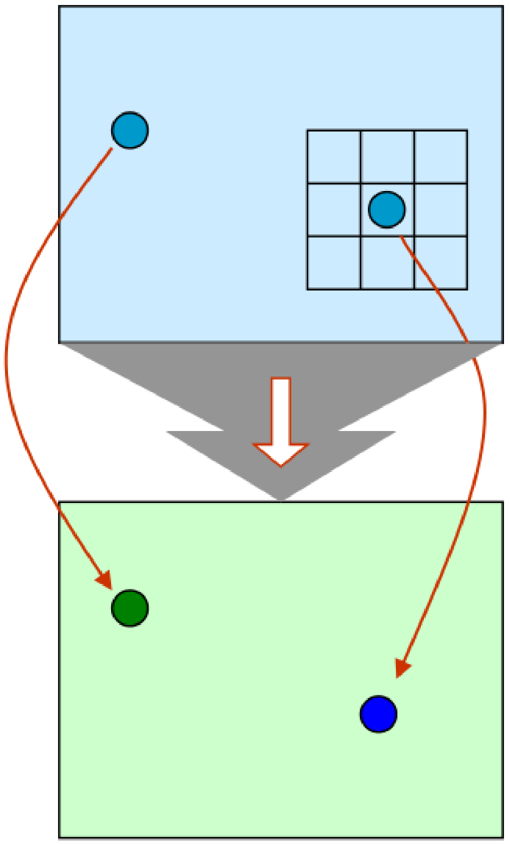
Example

Fig. MRI image

Operation Types

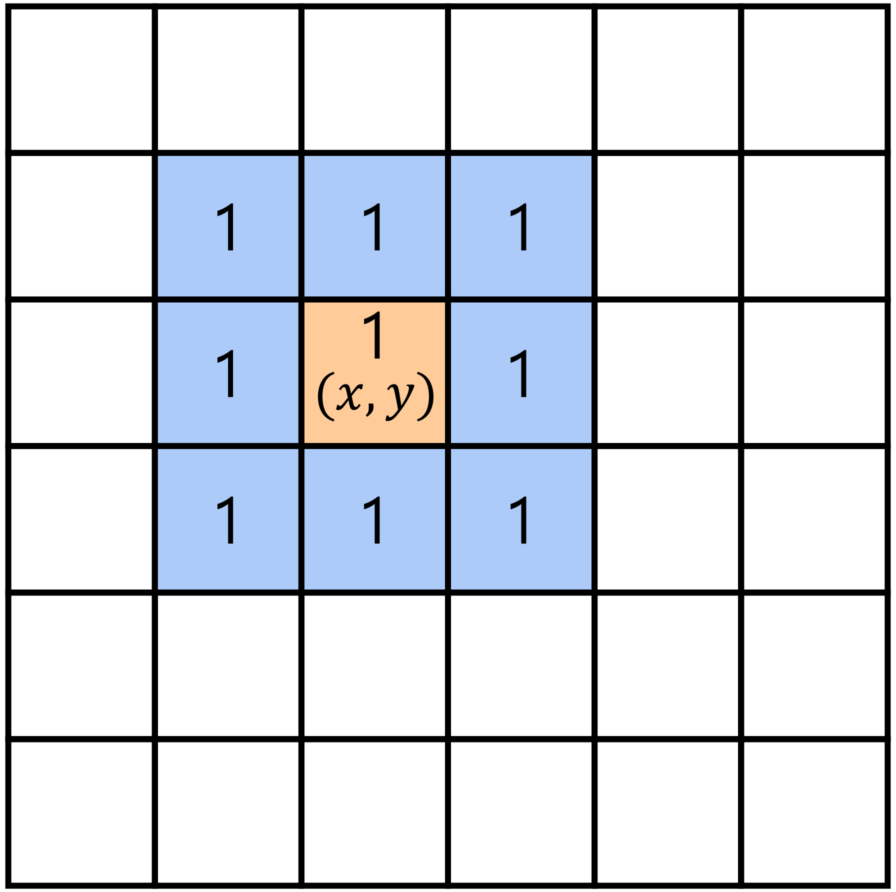
- Point Operation
  - Gray-level transformation
- Local Operation
  - Mask Processing or filtering
- Global Operation
  - Use values of all pixels
  - (e.g.) Fourier transform

    Histogram equalization, etc



Example 1

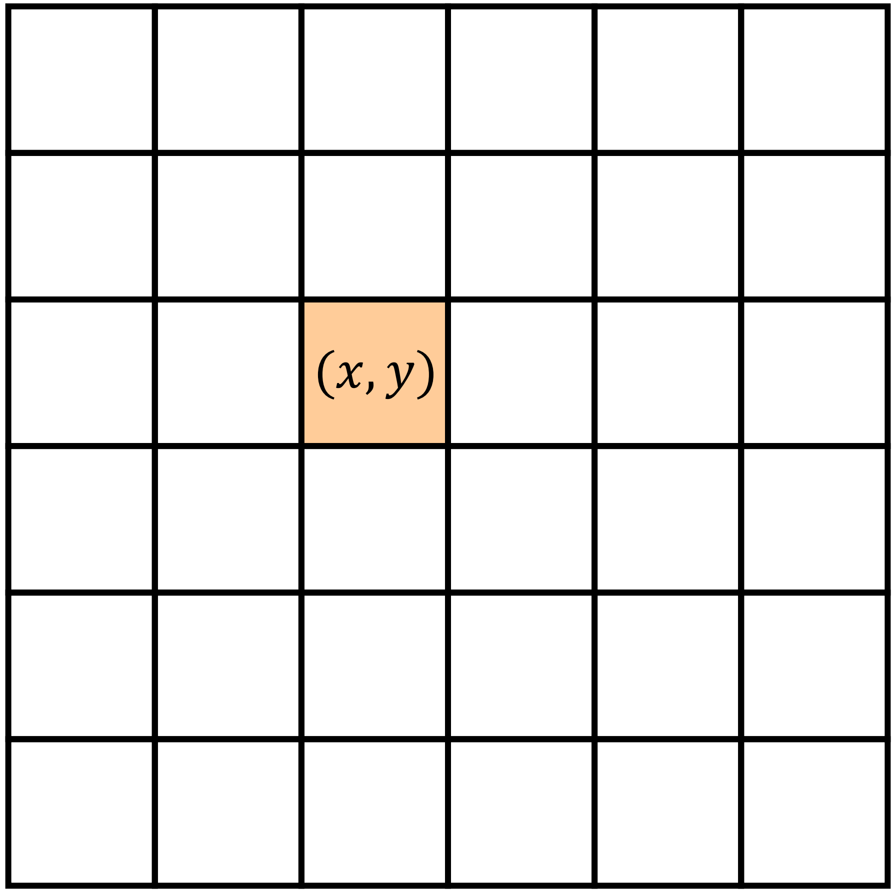
- $g(x, y) = T[f(x, y)]$
  - e.g. neighborhood is a 3×3 square
  - T: compute the average intensity of the neighborhood
  - then $g(x, y)$ = 1/9
  - This is called spatial filtering, and the 3×3 neighborhood, along with the operation is called a filter



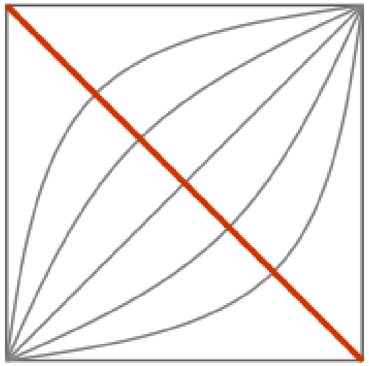
Example 2

- $g(x, y) = T[f(x, y)]$
  - e.g. neighborhood is a 1×1
  - Called intensity transformation
  - $s = T[r]$
  - r: intensity of input pixel

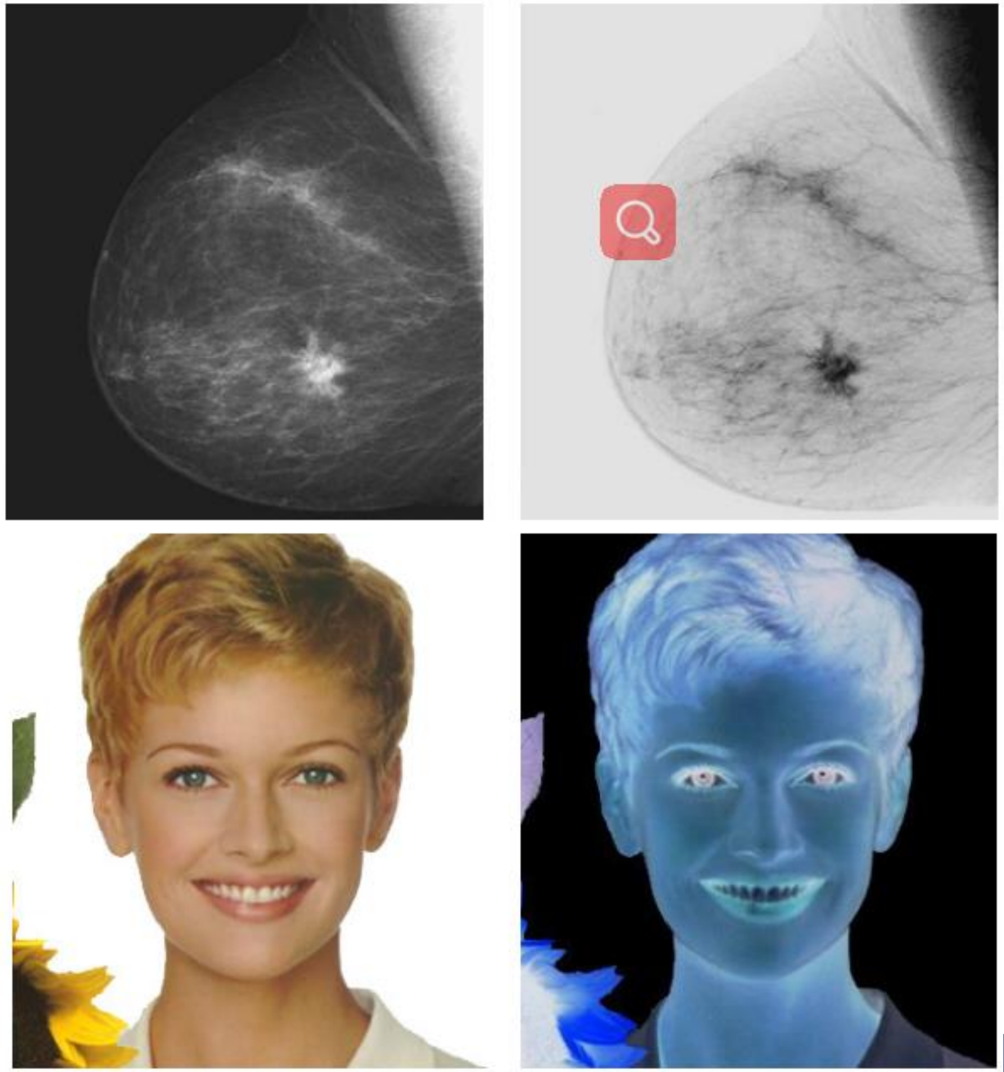
    s: intensity of output pixel



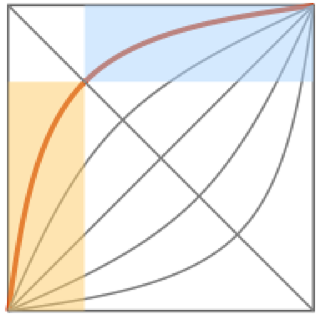
Intensity Transformation



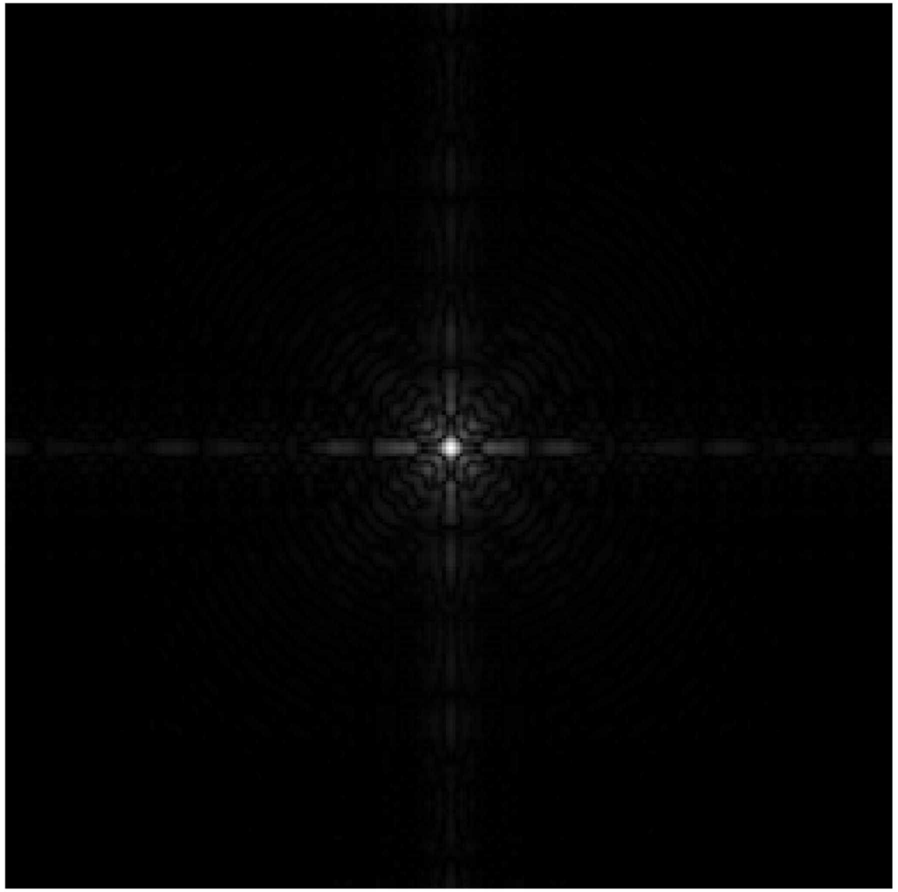
- Image Negatives: $s = L - 1 - r$

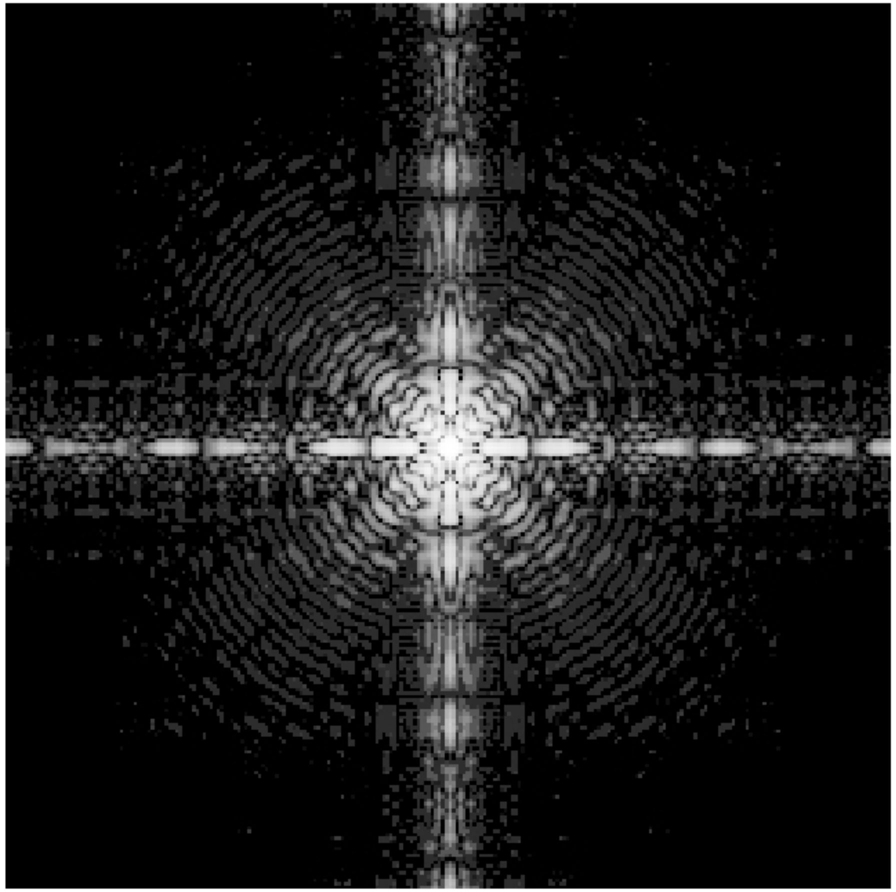


Intensity Transformation

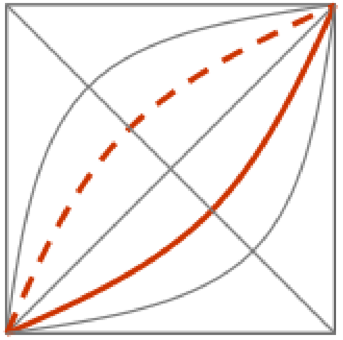


- Log Transformation: $s = c\log(1 + r)$

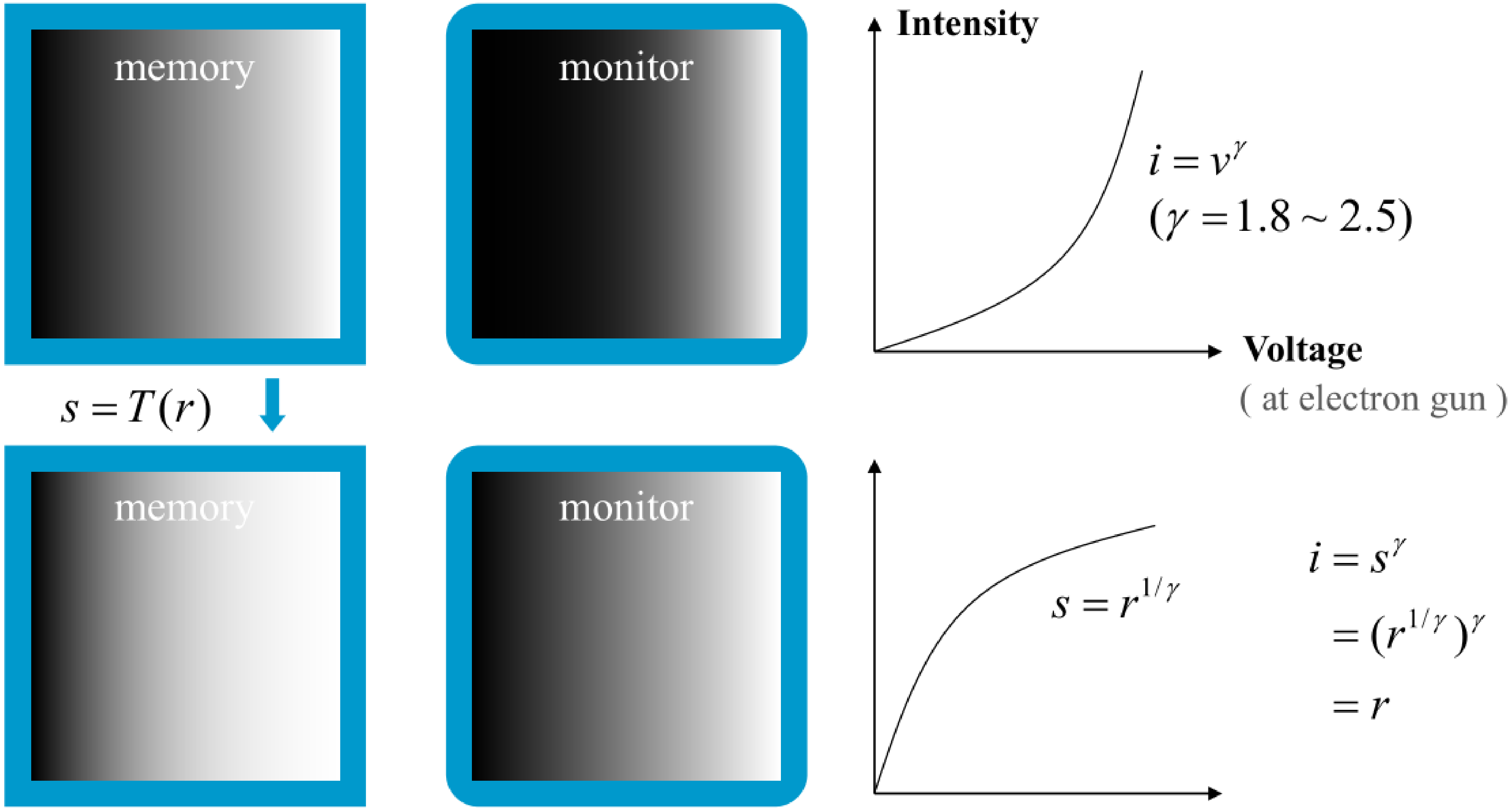




Intensity Transformation



- Power-Law Transformations: $s = cr^{\gamma}$

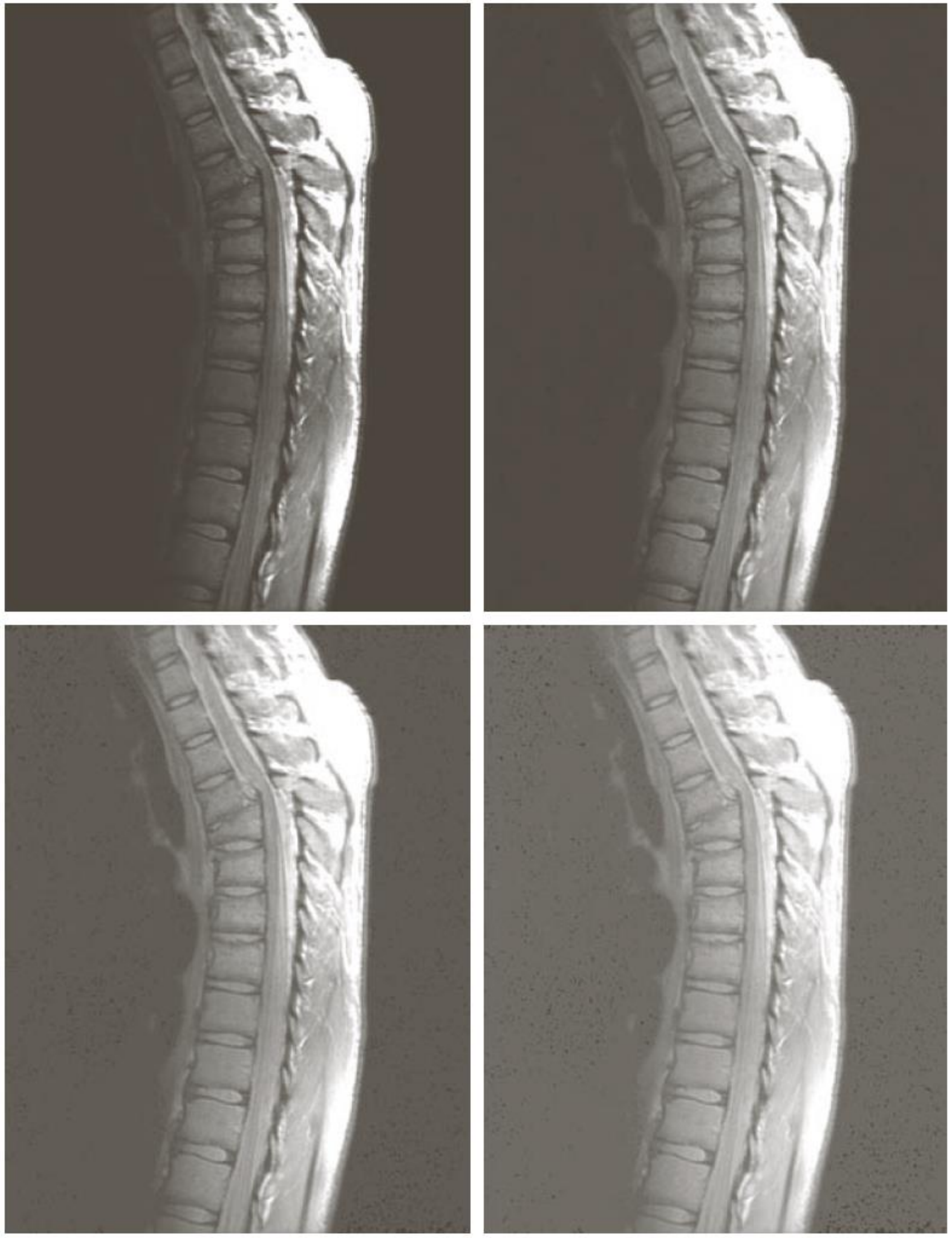


Intensity Trans

$s = cr^{\gamma}$

- Power-Law (G

Original MRI image



$c = 1, \gamma = 0.6$

$c = 1, \gamma = 0.4$

$c = 1, \gamma = 0.3$

Intensity Tra

$s = cr^{\gamma}$

- Power-Law

Original Aerial image

$c = 1, \gamma = 3.0$

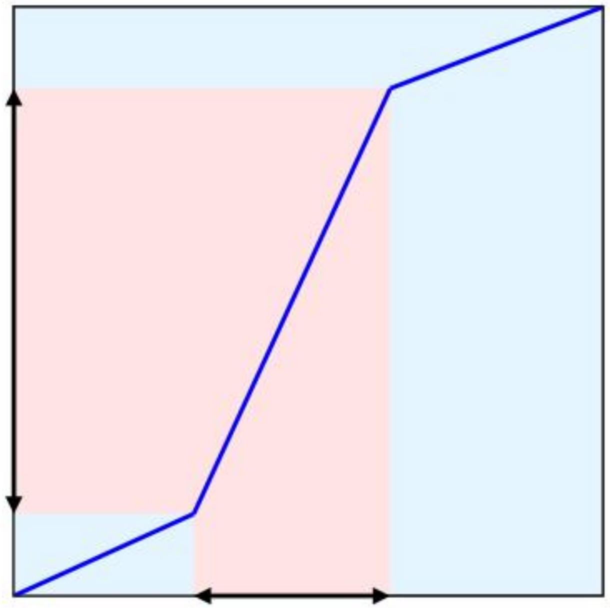
$c = 1, \gamma = 4.0$

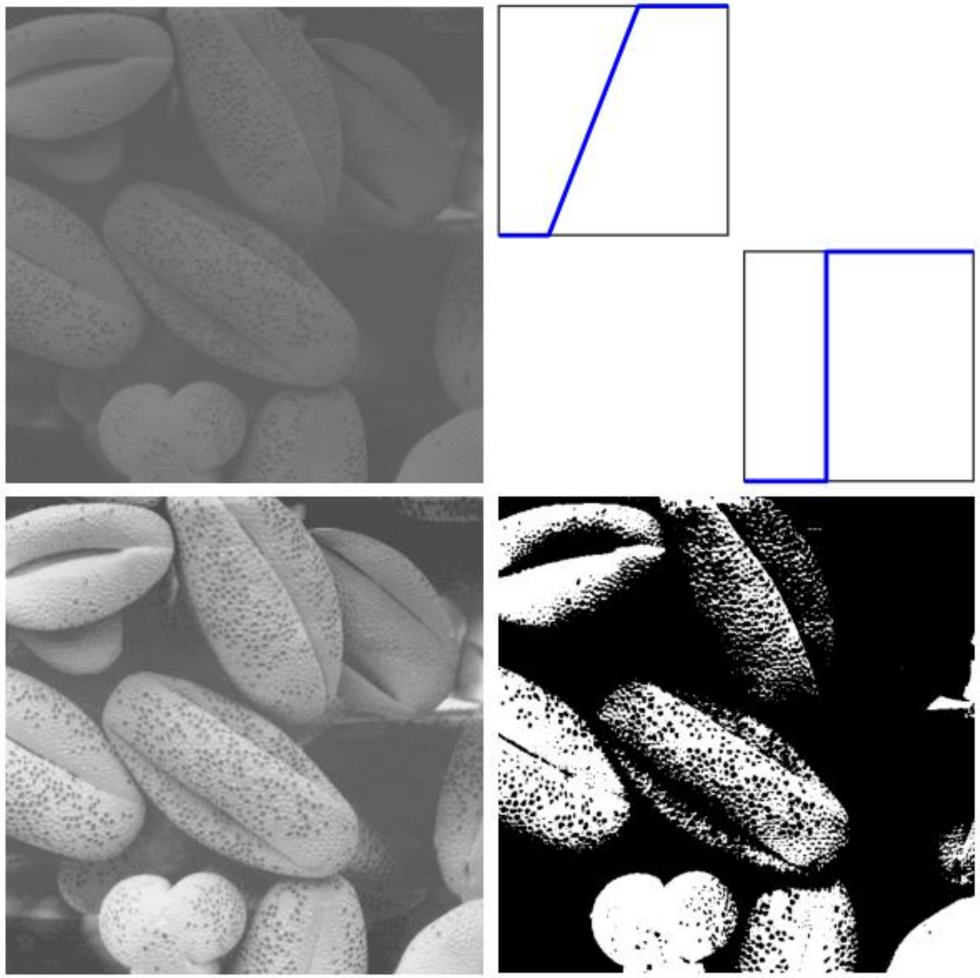
$c = 1, \gamma = 5.0$

Intensity Transformation

❑ Piecewise-Linear Transformation Functions

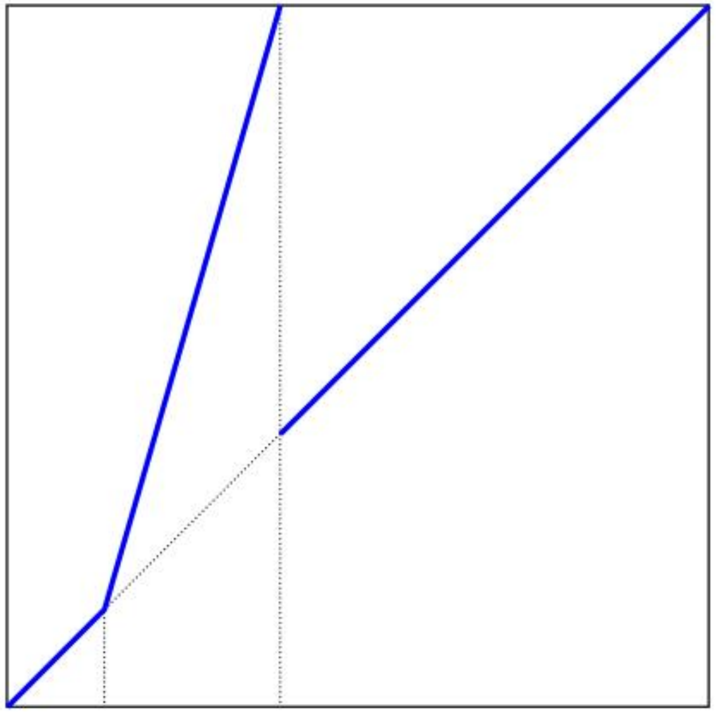
- Contrast Stretching





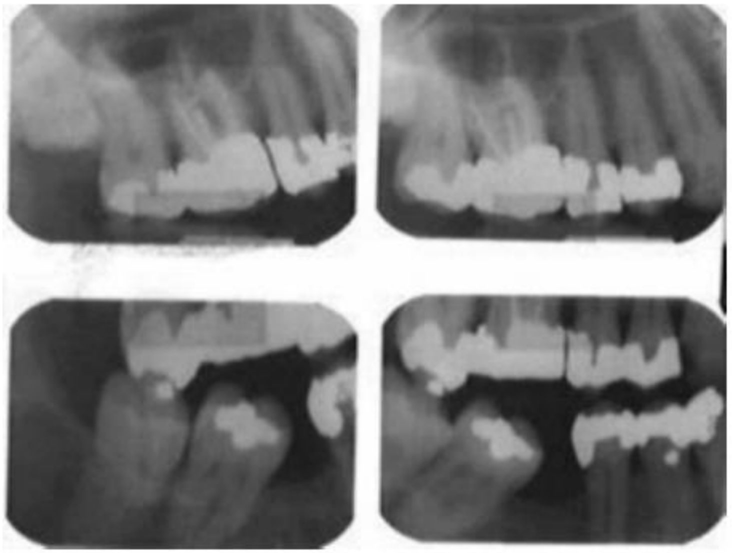
Intensity Transformation

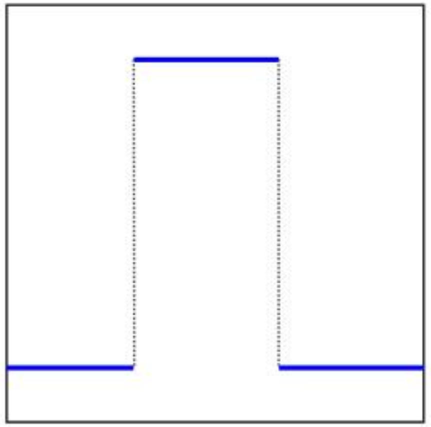
- Gray-Level Stretching

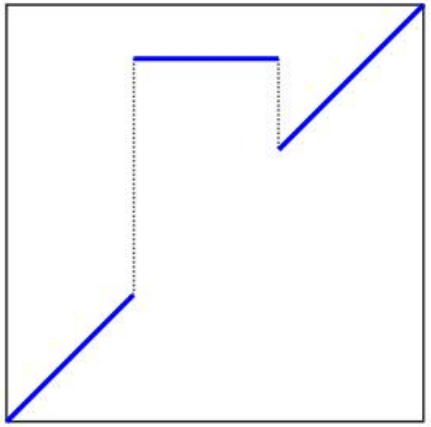


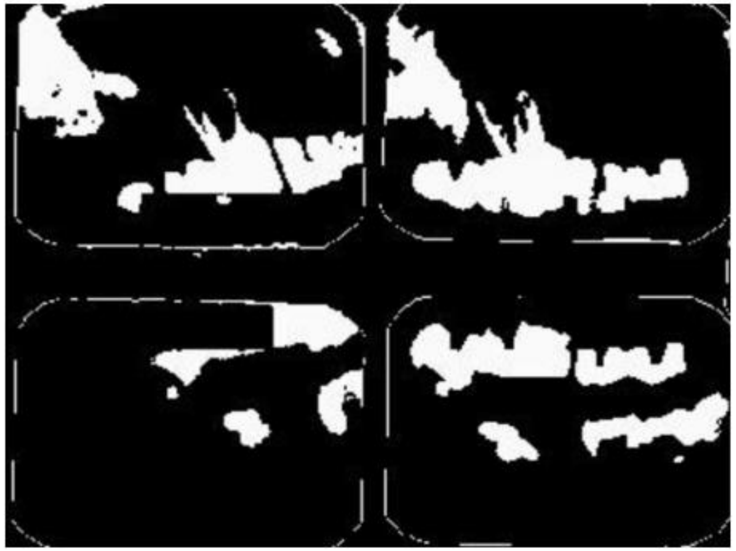
Intensity Transformation

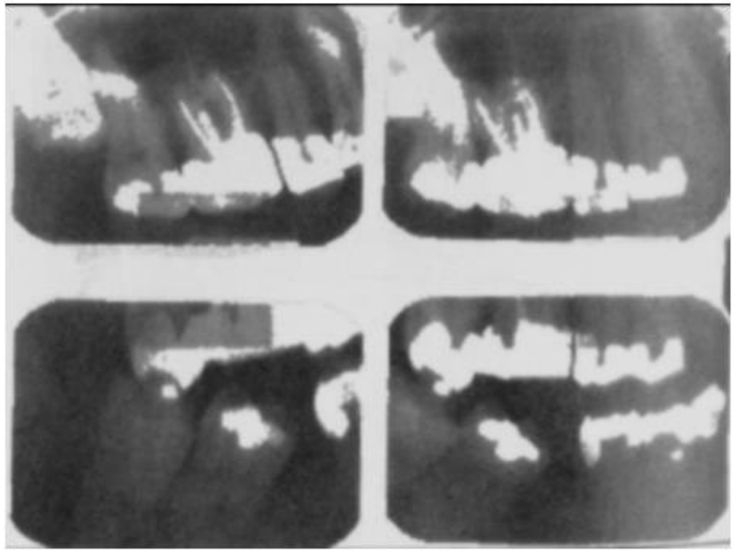
- Gray-Level Slicing





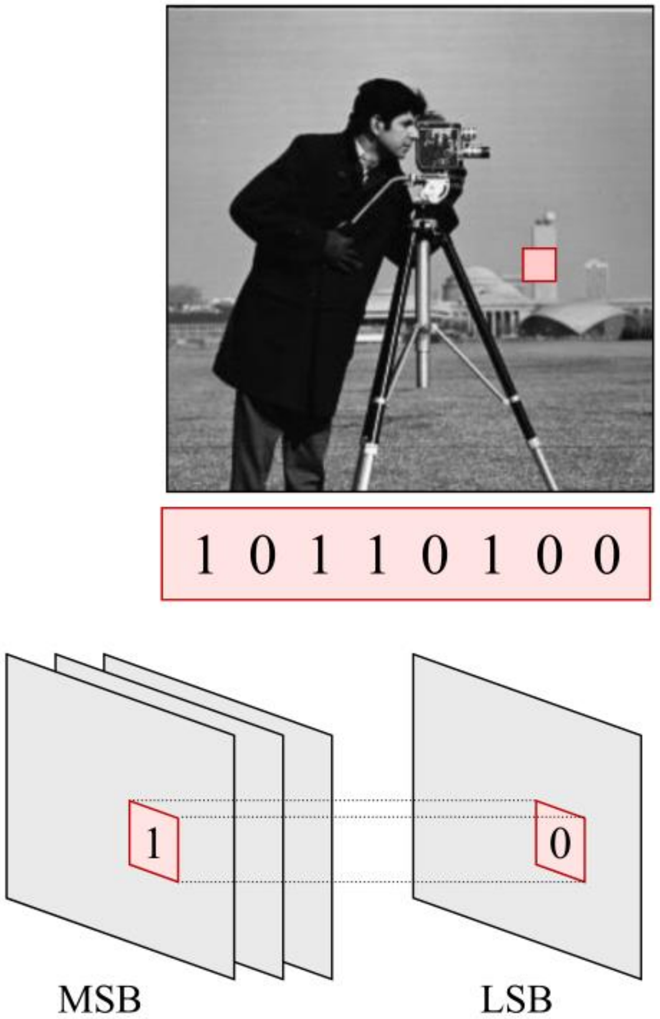


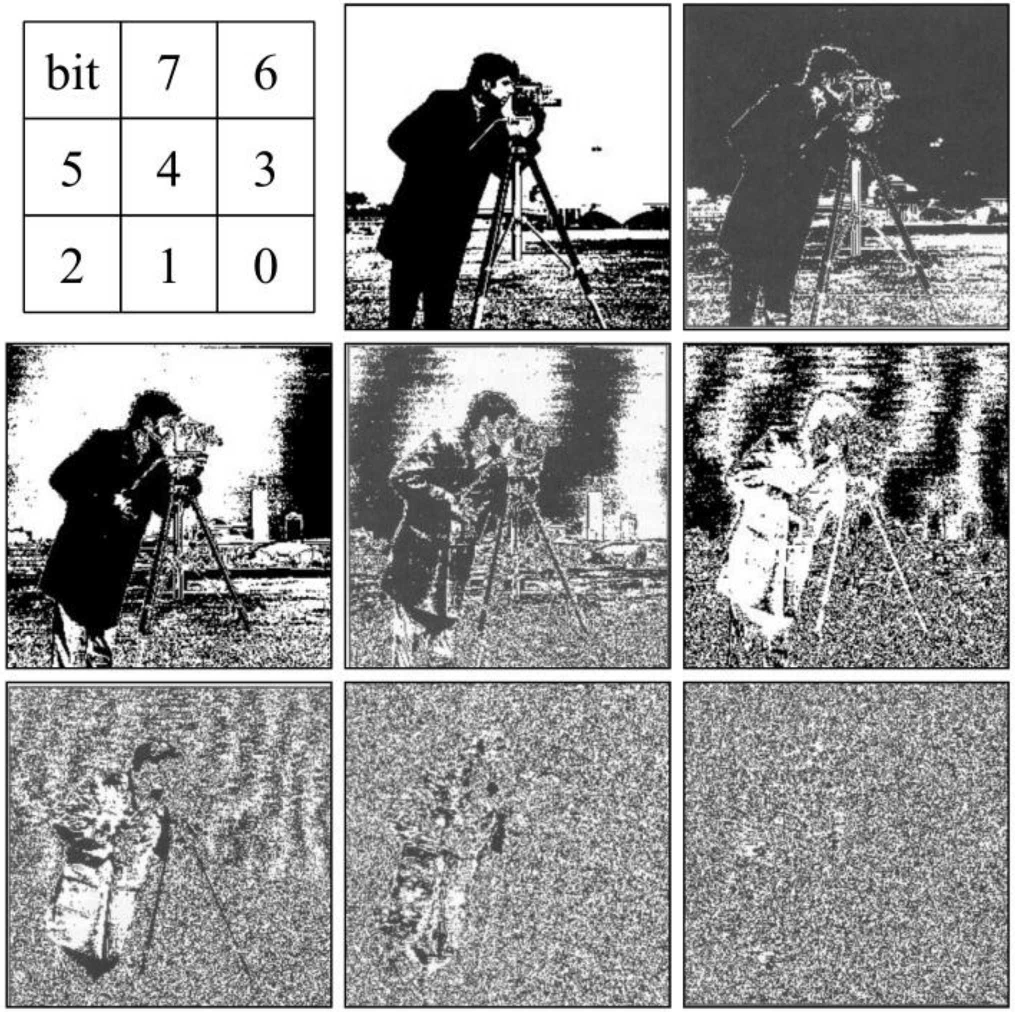




Intensity Transformation

- Bit-Plane Slicing





Histogram Processing

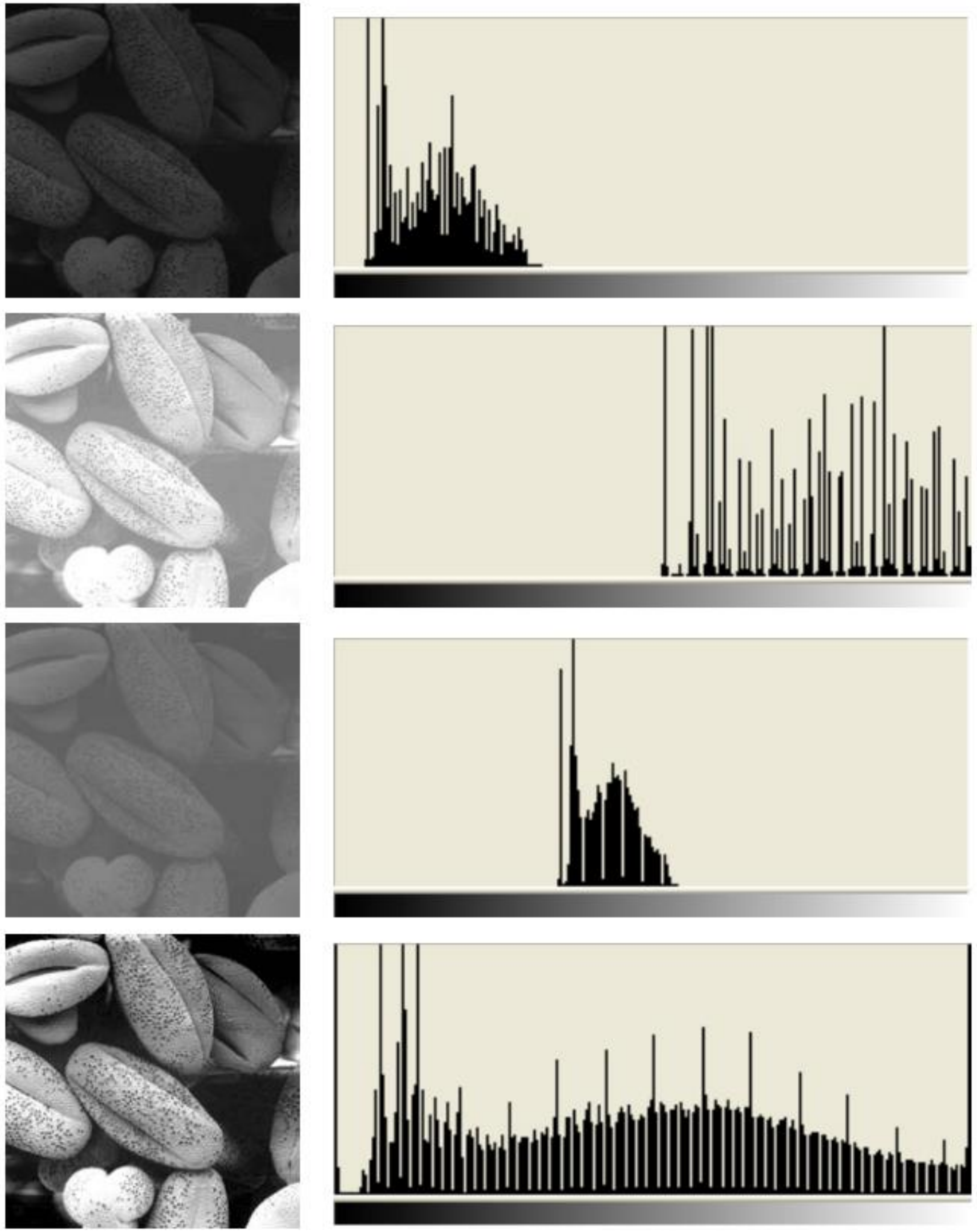
- Histogram

$$h(r_k) = n_k$$

$$k = 0, 1, \cdots, L-1$$

$$p(r_k) = \frac{n_k}{\sum_k n_k}$$

$$\sum_k p(r_k) = 1$$



❑ Background (option)

- Cumulative Distribution Function (CDF), $F(x)$ for a continuous random variable X

$$F(X) = P(X \leq x)$$

1. $F(-\infty) = 0$
2. $F(\infty) = 1$
3. $0 \leq F(x) \leq 1$
4. $F(x_1) \leq F(x_2) \; if \; x_1 \leq x_2$
5. $P(x_1 < x \leq x_2) = F(x_2) - F(x_1)$

- Probability Distribution Function (PDF), $p(x)$ for a continuous random variable x

$$p(x) = \frac{dF(x)}{dx}$$

1. $p(x) \geq 0$ for all x
2. $\int_{-\infty}^{\infty} p(x)dx = 1$
3. $F(x) = \int_{-\infty}^{x} p(\alpha)d\alpha$, where α is a dummy variable
4. $P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} p(x)dx$

❑ Transform of PDF using a monotonic function T(x)

$$y = T(x), 0 \le x \le 1$$

We assume that

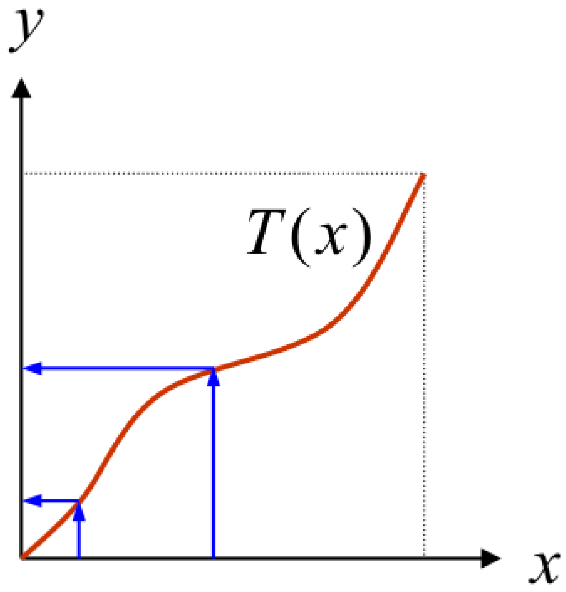
(a) $T(x)$ is a single-valued and monotonically increasing in the interval

$0 \le x \le 1$; and

(b) $0 \le T(x) \le 1$ for $0 \le x \le 1$

$f(x) \le f(y)$ for $x \le y$
non − decreasing

$f(x) < f(y)$ for $x < y$
strictly increasing ⇒ one-to-one



- The inverse transformation is denoted

$$x = T^{-1}(y), 0 \leq y \leq 1$$

If $T^{-1}(y)$ is single-valued and non-decreasing, then

$$p_t(y) = p_o(x)\left|\frac{dx}{dy}\right|$$

If $y = T(x) = \int_0^x p_o(\alpha)\, d\alpha,$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\int_0^x p_o(\alpha) d\alpha\right] = p_o(x).$$ Leibniz's rule

$$\therefore\ p_t(y) = p_o(x)\left|\frac{dx}{dy}\right| = p_o(x)\left|\frac{1}{p_o(x)}\right| = 1$$

∴ Uniform PDF

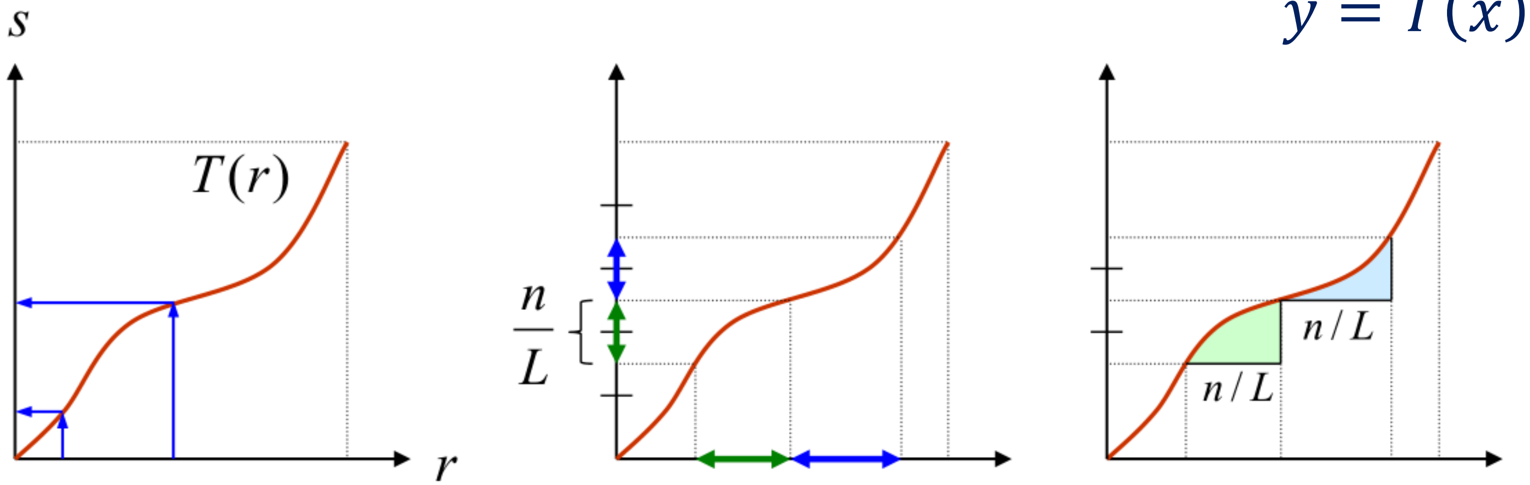
❑ Histogram Equalization

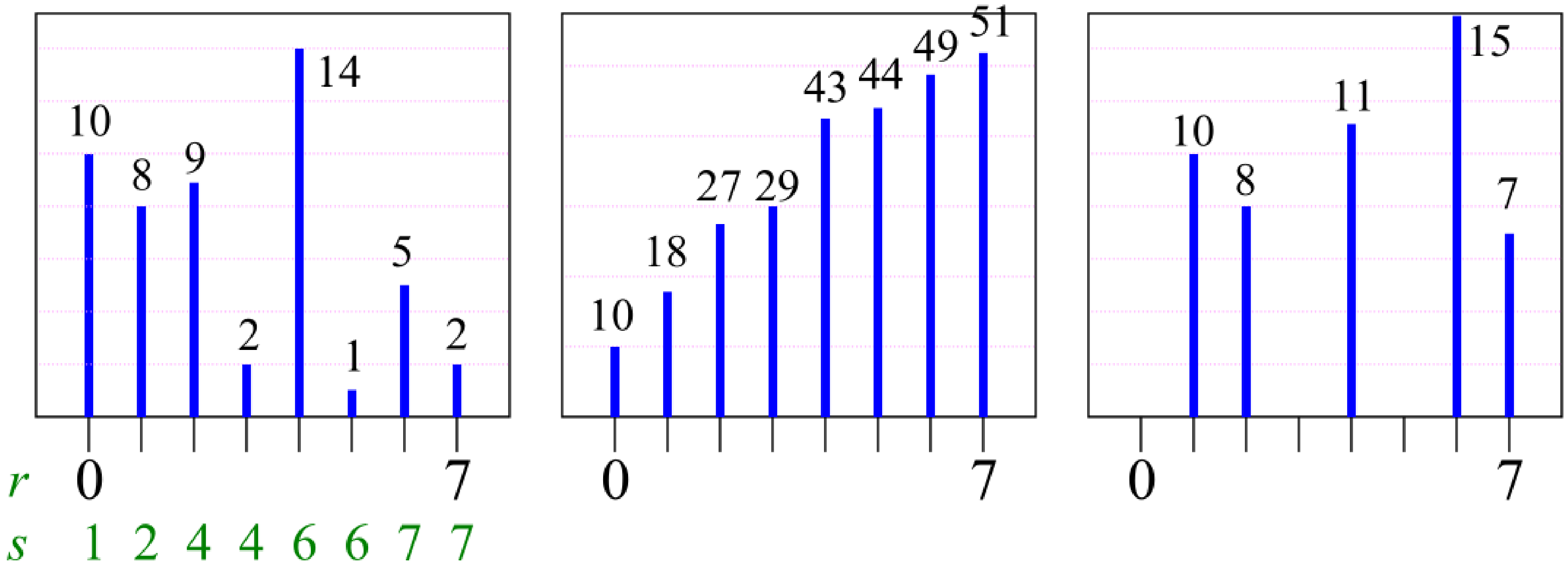
- Discrete version of transform of PDF

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \qquad n = \sum_{i=0}^{L-1} n_i$$

- Implementation

$$y = T(x) = \int_0^x p_x(\alpha)\, d\alpha$$





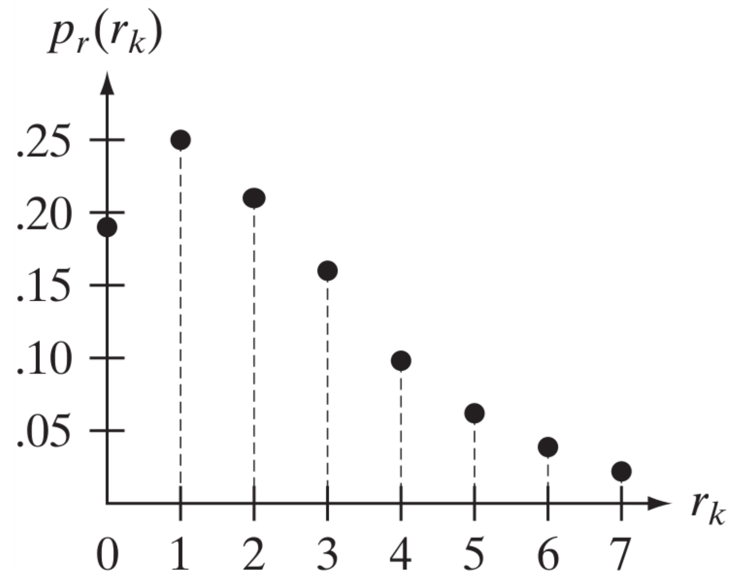
$$\frac{(10,\ 18,\ 27,\ 29,\ 43,\ 44,\ 49,\ 51)}{51} \times 7$$

$$\approx (1.37,\ 2.47,\ 3.71,\ 3.98,\ 5.90,\ 6.04,\ 6.73,\ 7.00)$$

$$\approx (\underline{1,\ 2,\ 4,\ 4,\ 6,\ 6,\ 7,\ 7})$$

❑ Example

Suppose that a 3-bit image of size pixels has the intensity distribution shown in the Table, where the intensity levels are integers in the range [0, L-1] = [0, 7].



r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

❑ Example

Using

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L-1$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
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$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

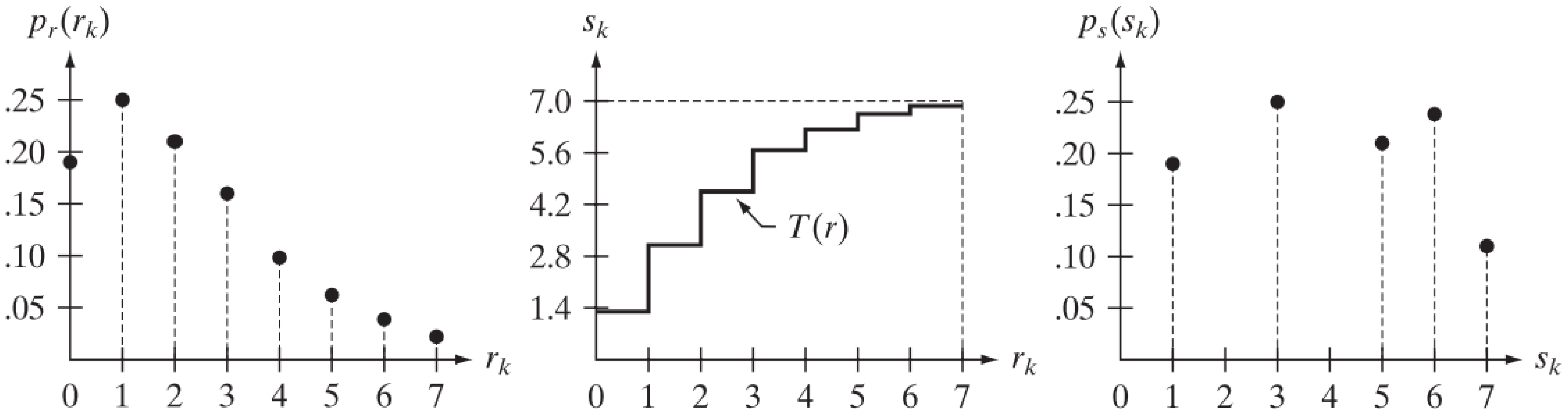
$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

$$s_2 = 4.55,\ s_3 = 5.67,\ s_4 = 6.23,\ s_5 = 6.65,\ s_6 = 6.86,\ s_7 = 7.00.$$

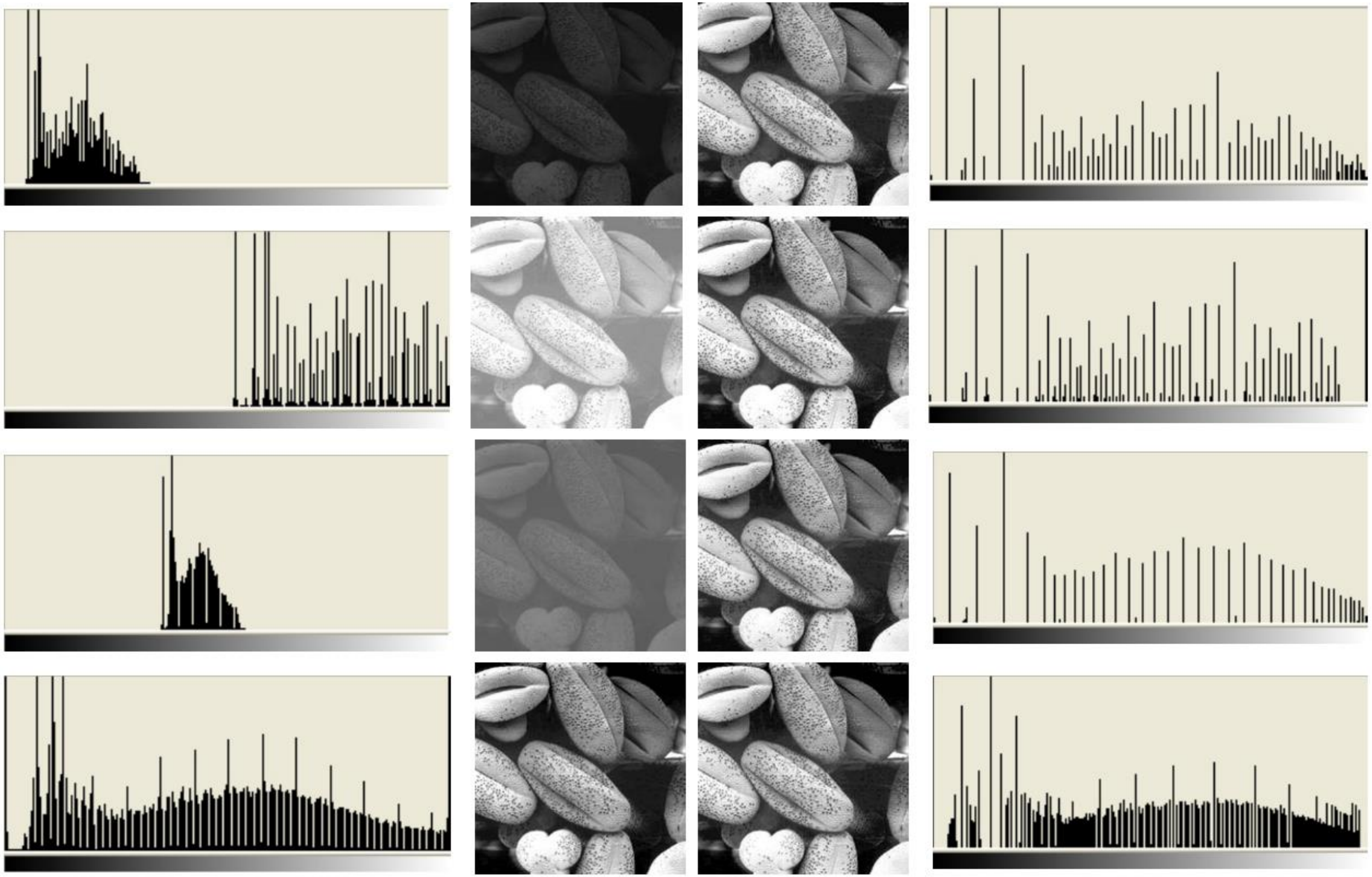
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L-1$$

$s_0 = 1.33 \rightarrow 1$	$s_4 = 6.23 \rightarrow 6$
$s_1 = 3.08 \rightarrow 3$	$s_5 = 6.65 \rightarrow 7$
$s_2 = 4.55 \rightarrow 5$	$s_6 = 6.86 \rightarrow 7$
$s_3 = 5.67 \rightarrow 6$	$s_7 = 7.00 \rightarrow 7$



Histogram Processing

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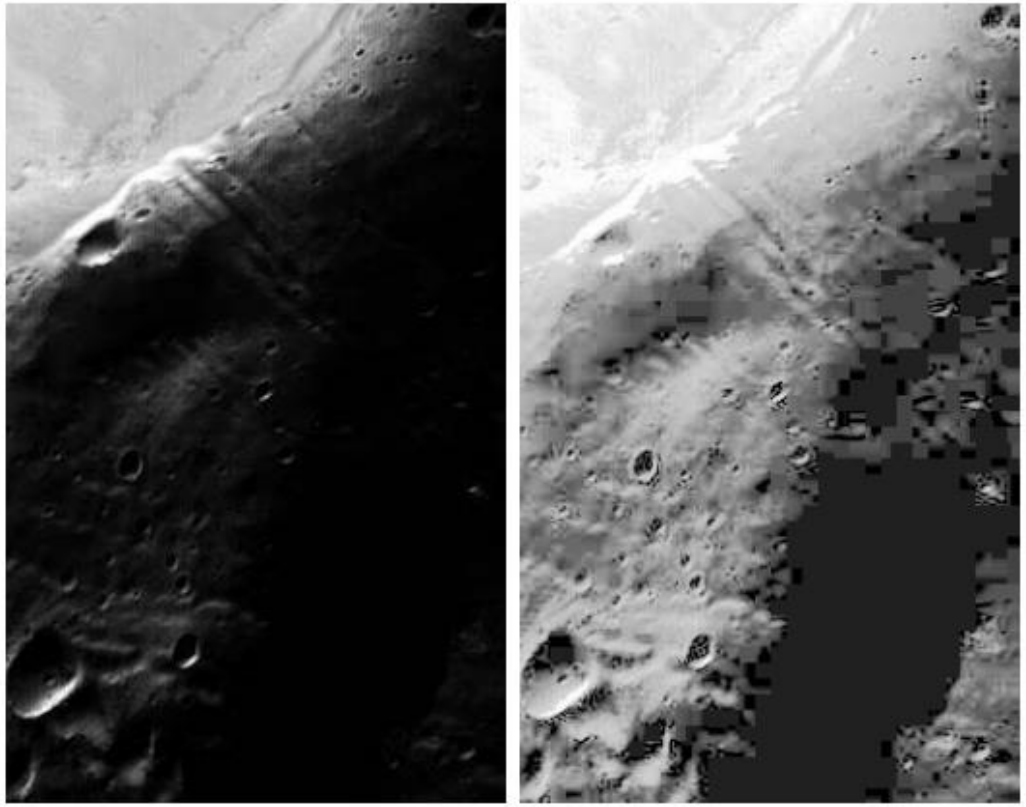
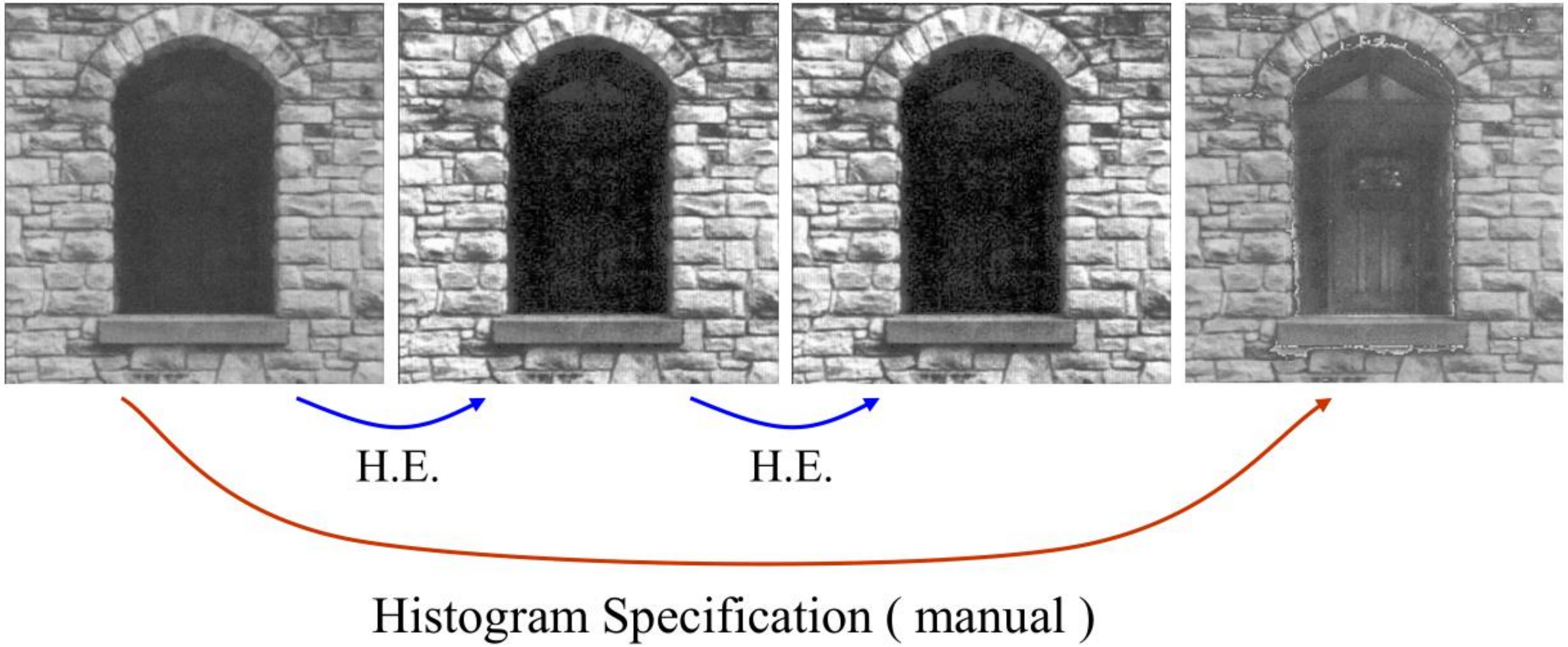


❑ Histogram Specification

- Histogram equalization is an automatic process
- Repeat appliance of H.E. is useless



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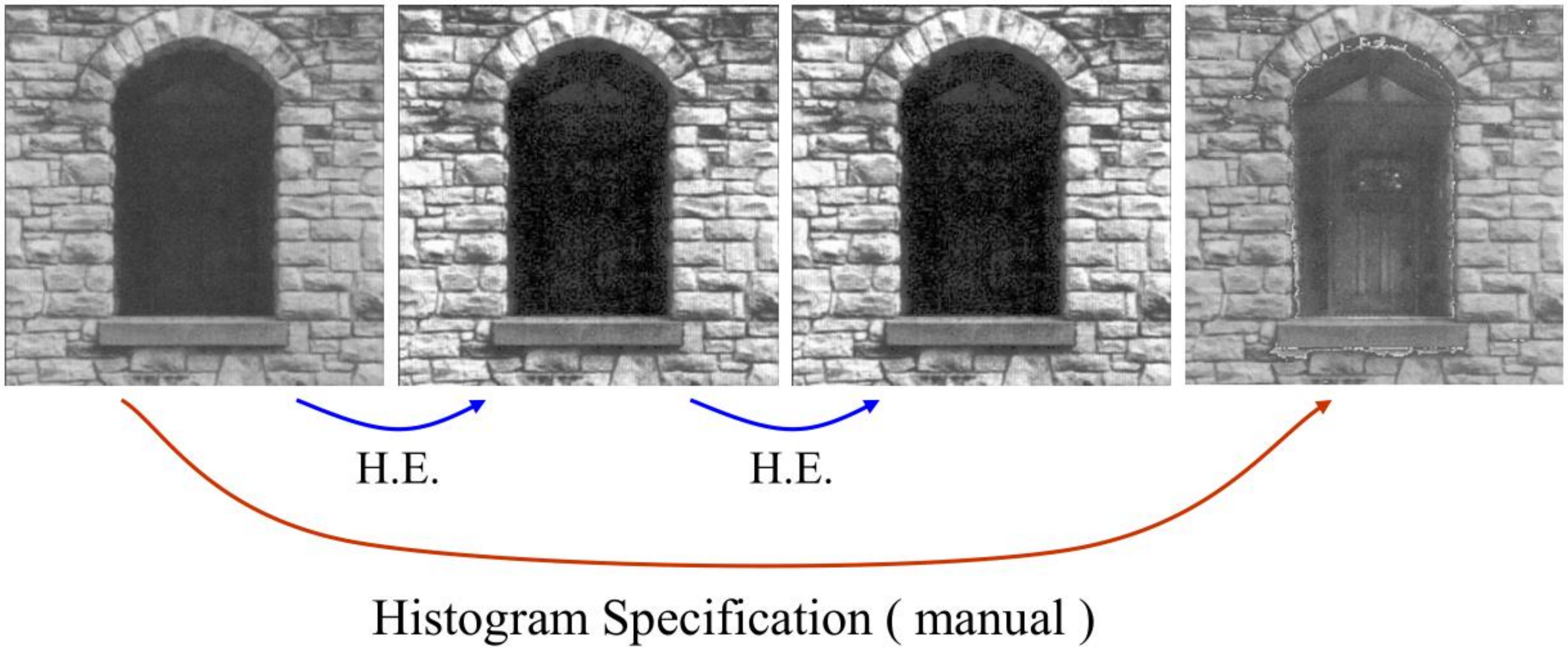


❑ Histogram Specification

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

Histogram equalization

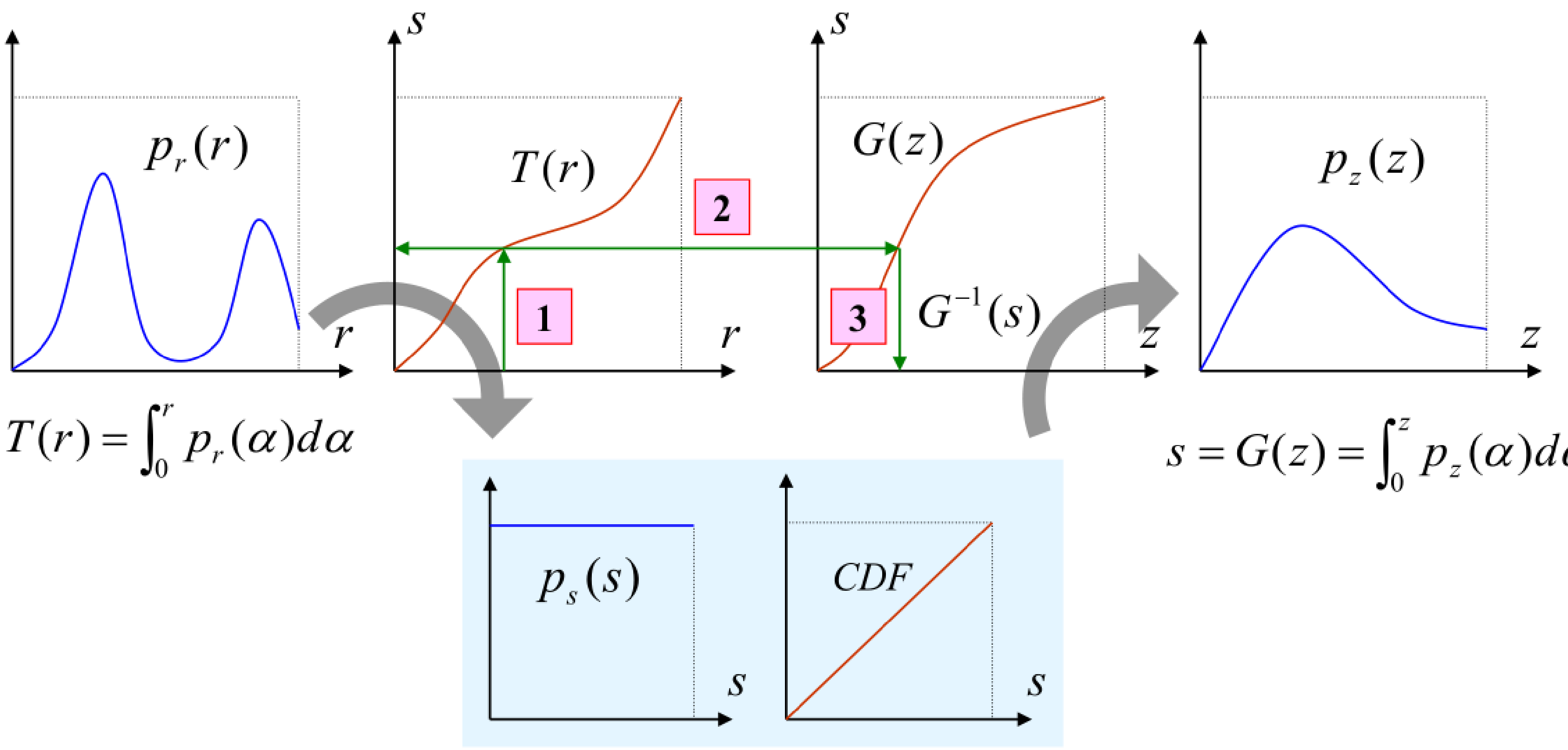
r: input image intensity

z: desired image intensity

$$z = G^{-1}[T(r)] = G^{-1}(s)$$

❑ Histogram Specification

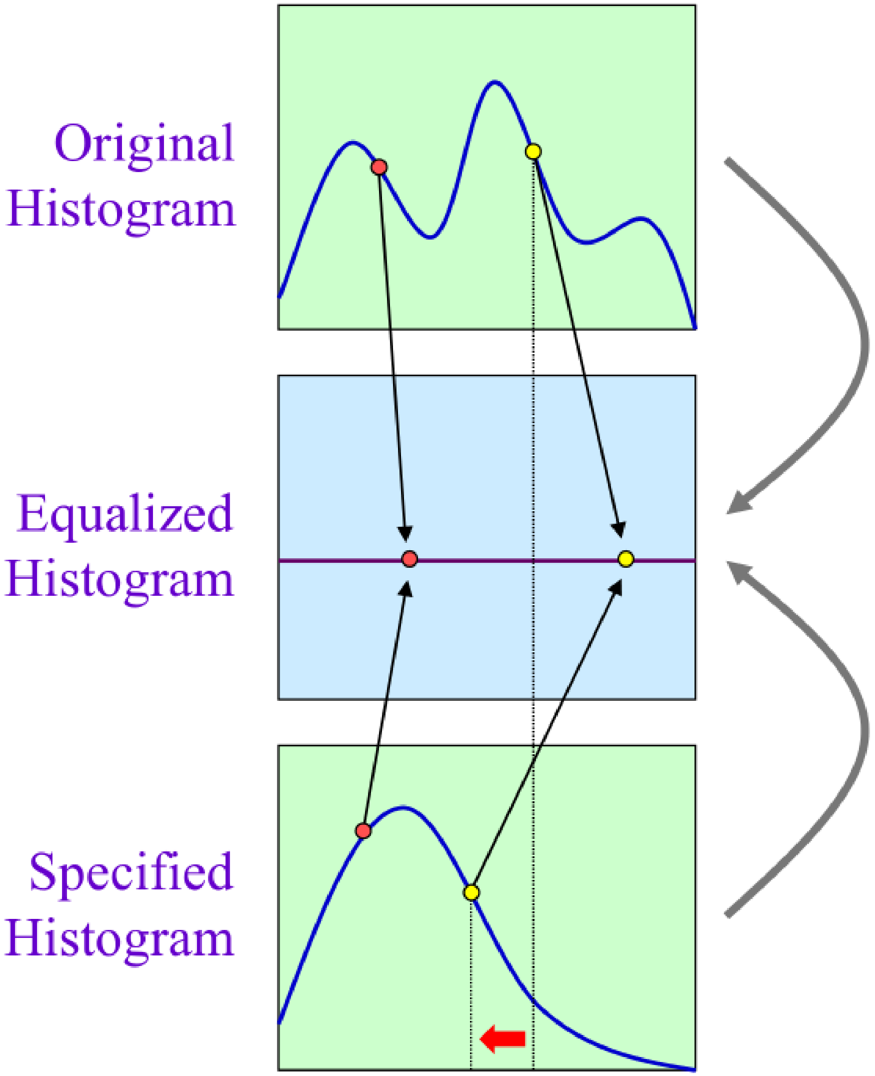
$$z = G^{-1}(s) = G^{-1}[T(r)]$$



$$s = T(r) = \int_0^r p_r(\alpha)d\alpha$$

$$s = G(z) = \int_0^z p_z(\alpha)d\alpha$$

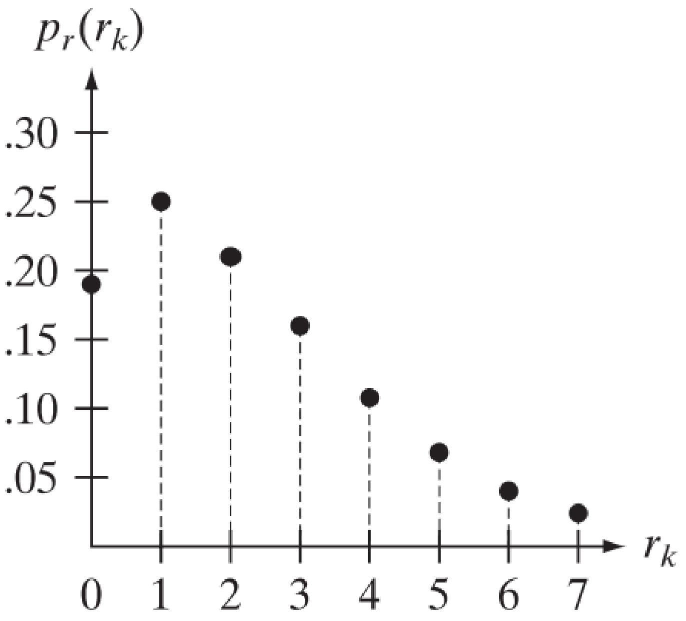
❑ Histogram Specification

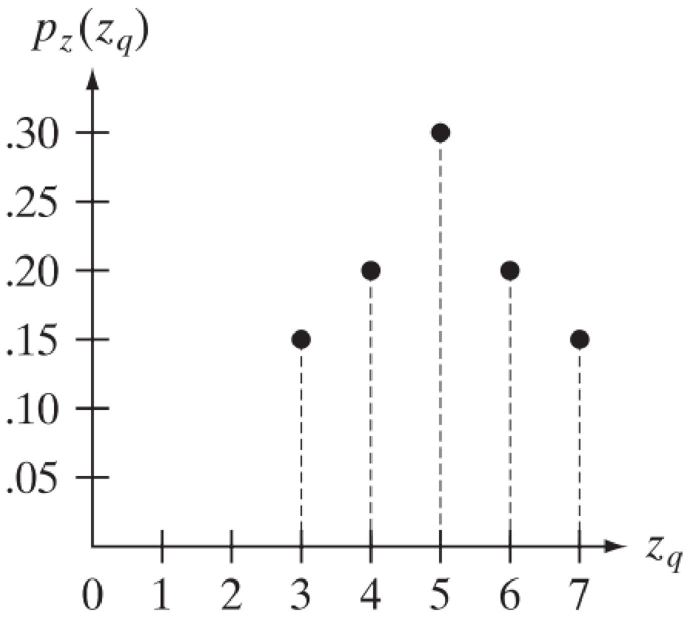


r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

❑ Example

Consider a 64 * 64 hypothetical image, whose histogram is shown in the Figure (a). It is desired to transform this histogram so that it will have the values specified in the second column of the table. Figure (b) shows a sketch of this histogram.





z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

- Example (cont'd)

a) Performing histogram equalization as in the previous example

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

$$s_2 = 4.55, s_3 = 5.67, s_4 = 6.23, s_5 = 6.65, s_6 = 6.86, s_7 = 7.00.$$

After rounding up,

$$s_0 = 1 \quad s_2 = 5 \quad s_4 = 7 \quad s_6 = 7$$

$$s_1 = 3 \quad s_3 = 6 \quad s_5 = 7 \quad s_7 = 7$$

❑ Example (cont'd)

b) Computing all the values of the transformation function, G, using

$$G(z_q) = (L - 1)\sum_{i=0}^{q} p_z(z_i)$$

we could get

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00$$

$$G(z_1) = 7\sum_{j=0}^{1} p_z(z_j) = 7\left[p(z_0) + p(z_1)\right] = 0.00$$

and

$$G(z_2) = 0.00 \quad G(z_4) = 2.45 \quad G(z_6) = 5.95$$

$$G(z_3) = 1.05 \quad G(z_5) = 4.55 \quad G(z_7) = 7.00$$

- Example (cont'd)

After rounding-up:

$$G(z_0) = 0.00 \rightarrow 0 \qquad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_5) = 4.55 \rightarrow 5$$

$$G(z_2) = 0.00 \rightarrow 0 \qquad G(z_6) = 5.95 \rightarrow 6$$

$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_7) = 7.00 \rightarrow 7$$

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00$$

$$G(z_1) = 7\sum_{j=0}^{1} p_z(z_j) = 7\left[p(z_0) + p(z_1)\right] = 0.00$$

$$G(z_2) = 0.00 \quad G(z_4) = 2.45 \quad G(z_6) = 5.95$$

$$G(z_3) = 1.05 \quad G(z_5) = 4.55 \quad G(z_7) = 7.00$$

❑ Example (cont'd)

c) Finding the smallest value of z_q so that the value $G(z_q)$ is the closest to s_k.

We do this for every value of s_k to create the required mappings from s to z. For example, s_0 = 1, and we see that $G(z_3)$ = 1, which is a perfect match in this case, so we have the correspondence $s_0 \rightarrow z_3$.

s_k	$G(z_q)$	z_q
$s_0 = 1$	0	$z_0 = 0$
$s_1 = 3$	0	$z_1 = 1$
$s_2 = 5$	0	$z_2 = 2$
$s_3 = 6$	1	$z_3 = 3$
$s_4 = 7$	2	$z_4 = 4$
$s_5 = 7$	5	$z_5 = 5$
$s_6 = 7$	6	$z_6 = 6$
$s_7 = 7$	7	$z_7 = 7$

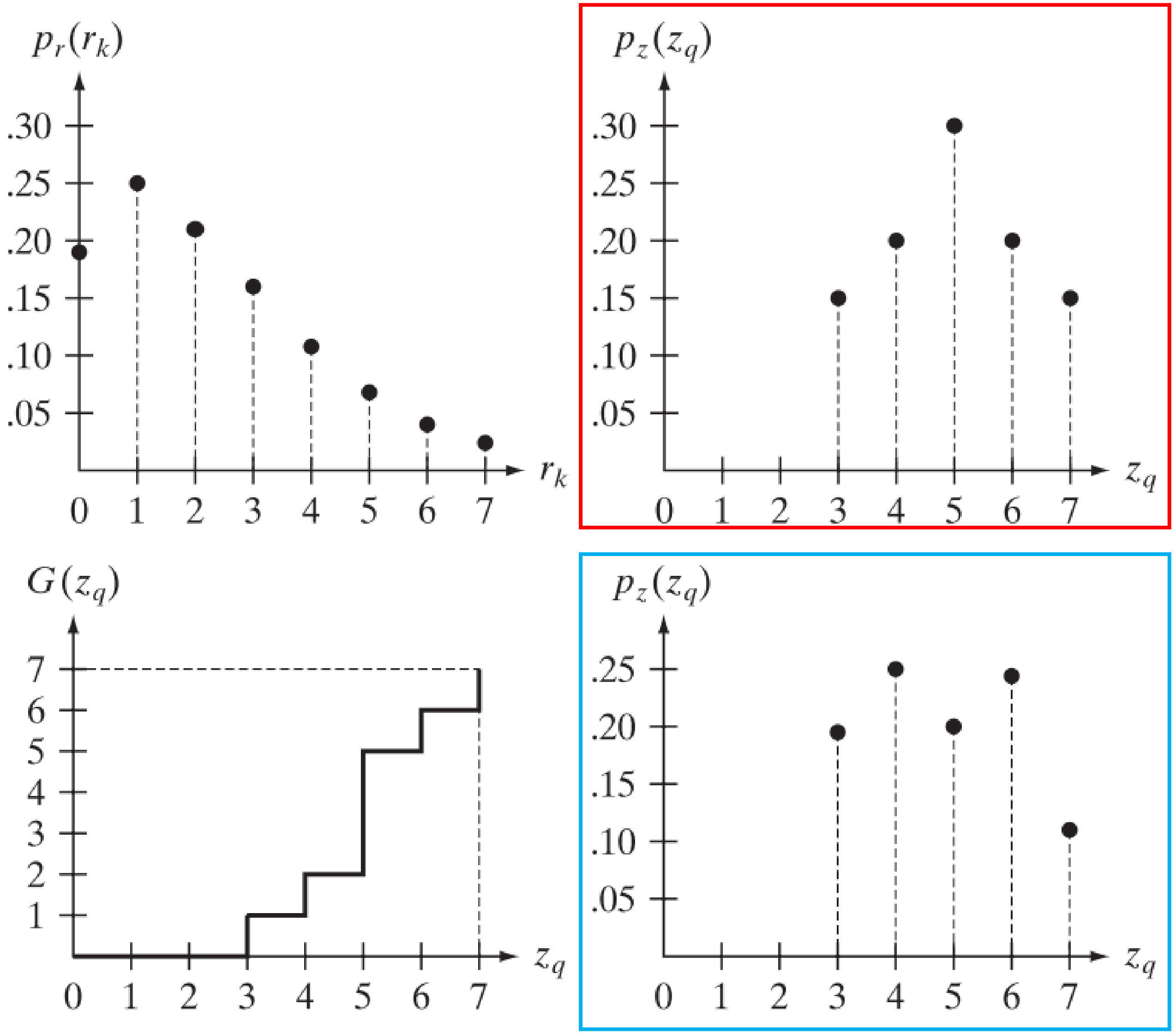
❑ Example (cont'd)

s_k	$G(z_q)$	z_q
$s_0 = 1$	0	$z_0 = 0$
$s_1 = 3$	0	$z_1 = 1$
$s_2 = 5$	0	$z_2 = 2$
$s_3 = 6$	1	$z_3 = 3$
$s_4 = 7$	2	$z_4 = 4$
$s_5 = 7$	5	$z_5 = 5$
$s_6 = 7$	6	$z_6 = 6$
$s_7 = 7$	7	$z_7 = 7$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

- Example (cont'd)

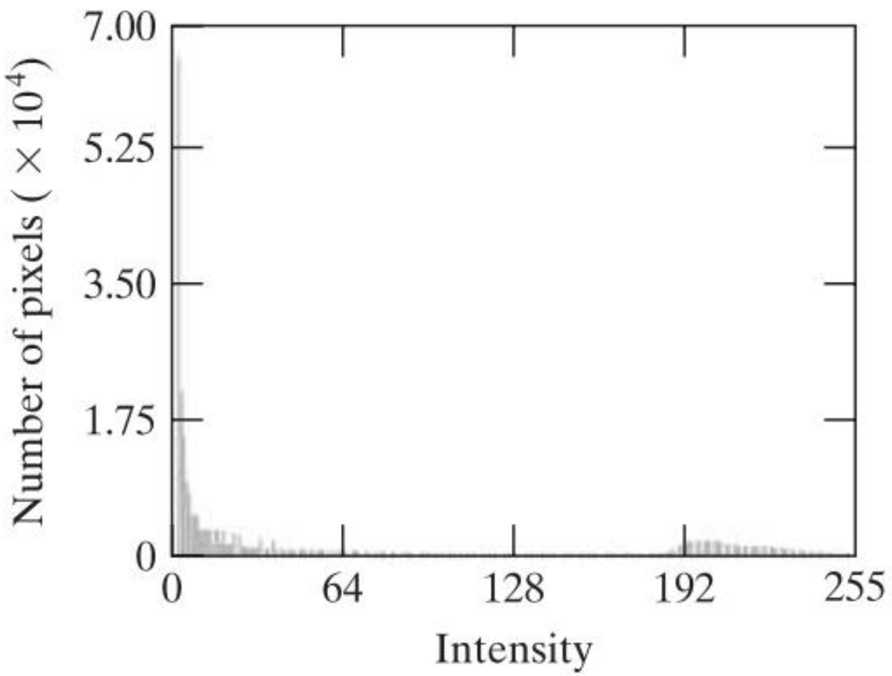
  Finally,

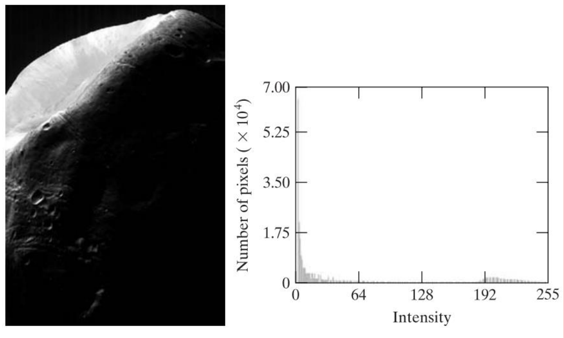


Specified histogram

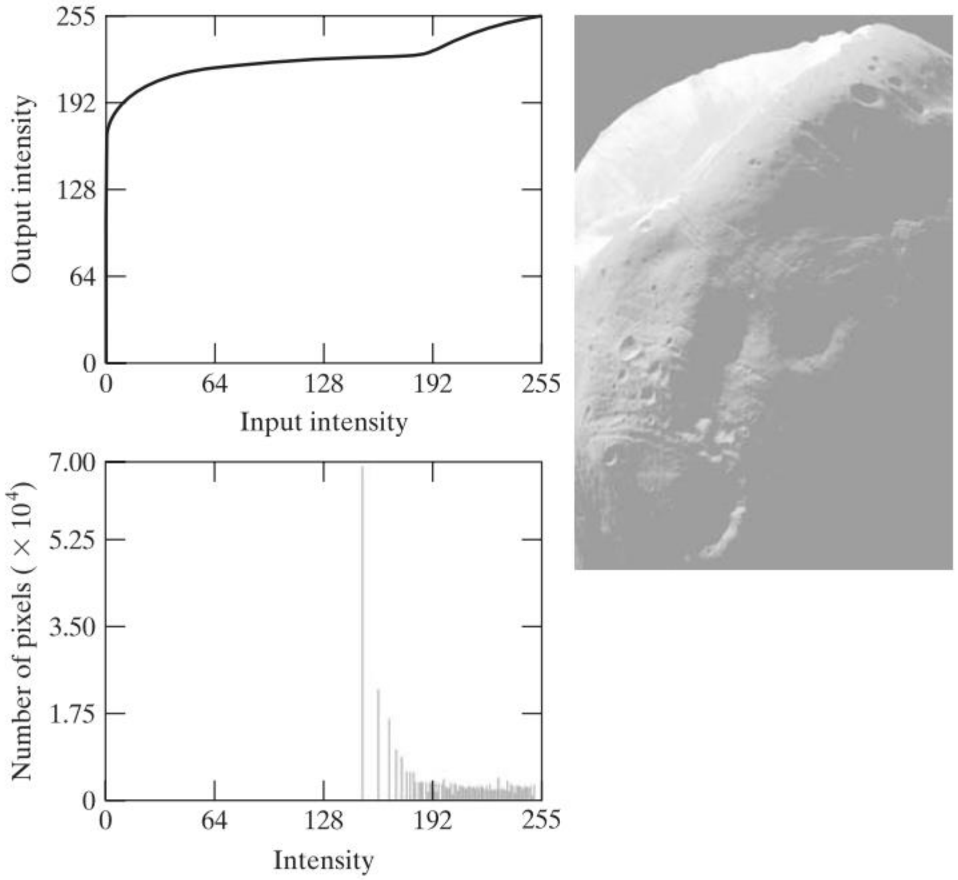
After performing histogram specification

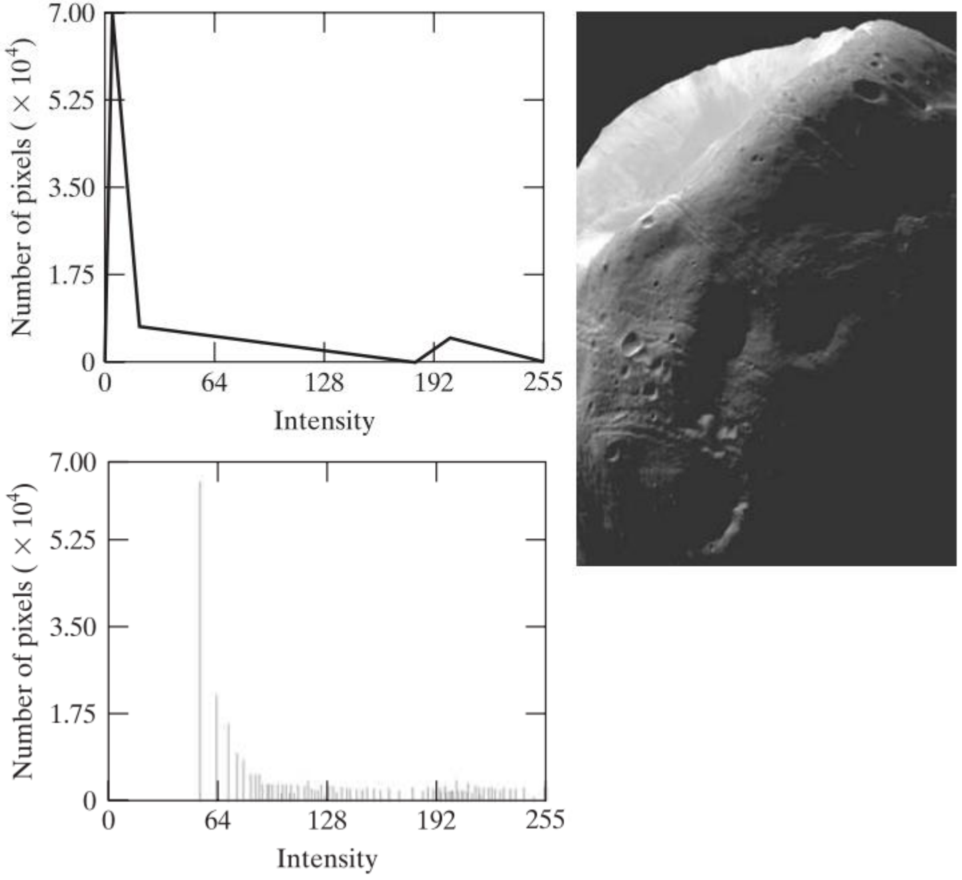
- Histogram equalization *v.s.* histogram specification





- Histogram equalization *v.s.* histogram specification





H.E.

H.S.

Summary

- Background
- Intensity Transformation
- Histogram Processing

Next

Lecture 3: Get Hand Dirty by Coding

Q & A

Thank You!