Lecture 4 Spatial Filtering

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Outline

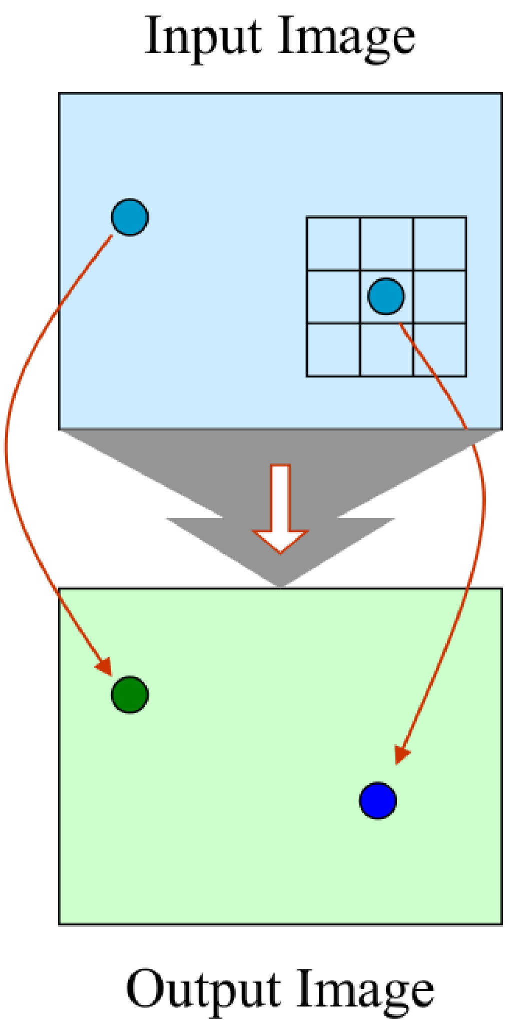
- Spatial Filtering
  - Fundamentals of Spatial Filtering
  - Smoothing Spatial Filters
  - Sharpening Spatial Filters

Review

❑ Operation Types

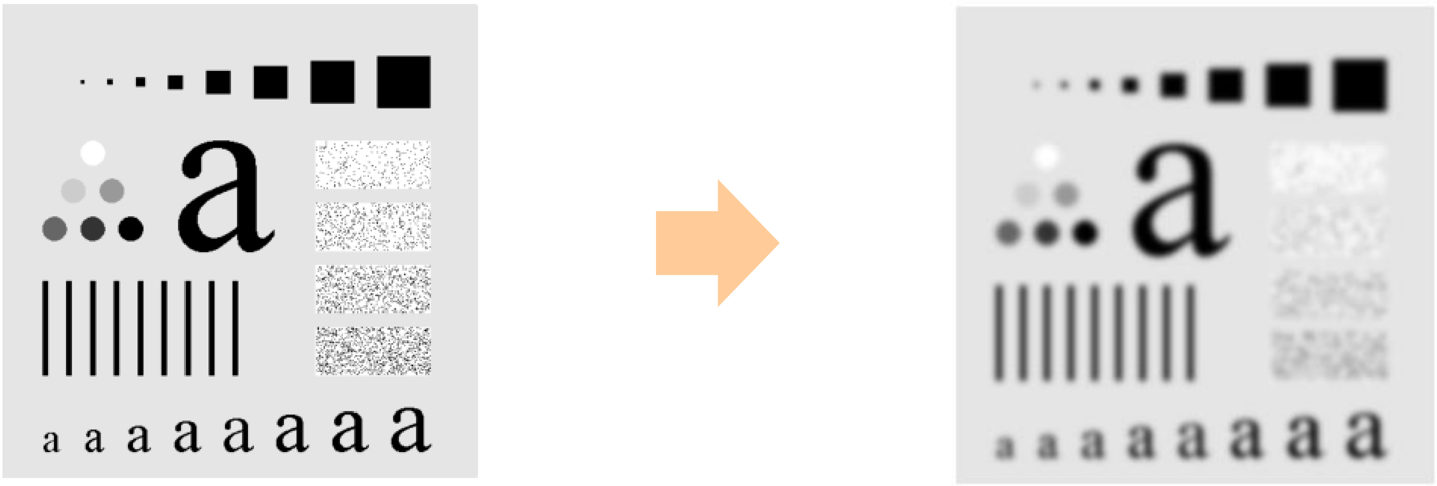
- ❖ Point Operation
  - Gray-level transformation
- ❖ Local Operation
  - Mask Processing or filtering
- ❖ Global Operation
  - Use values of all pixels
  - (e.g.) Fourier transform

    Histogram equalization, etc



Fundamentals of Spatial Filtering

- Filter
  - Borrowed from frequency domain processing
  - Accepting (passing) or rejecting certain frequency components.
- E.g. lowpass filter
  - blur (smooth) an image



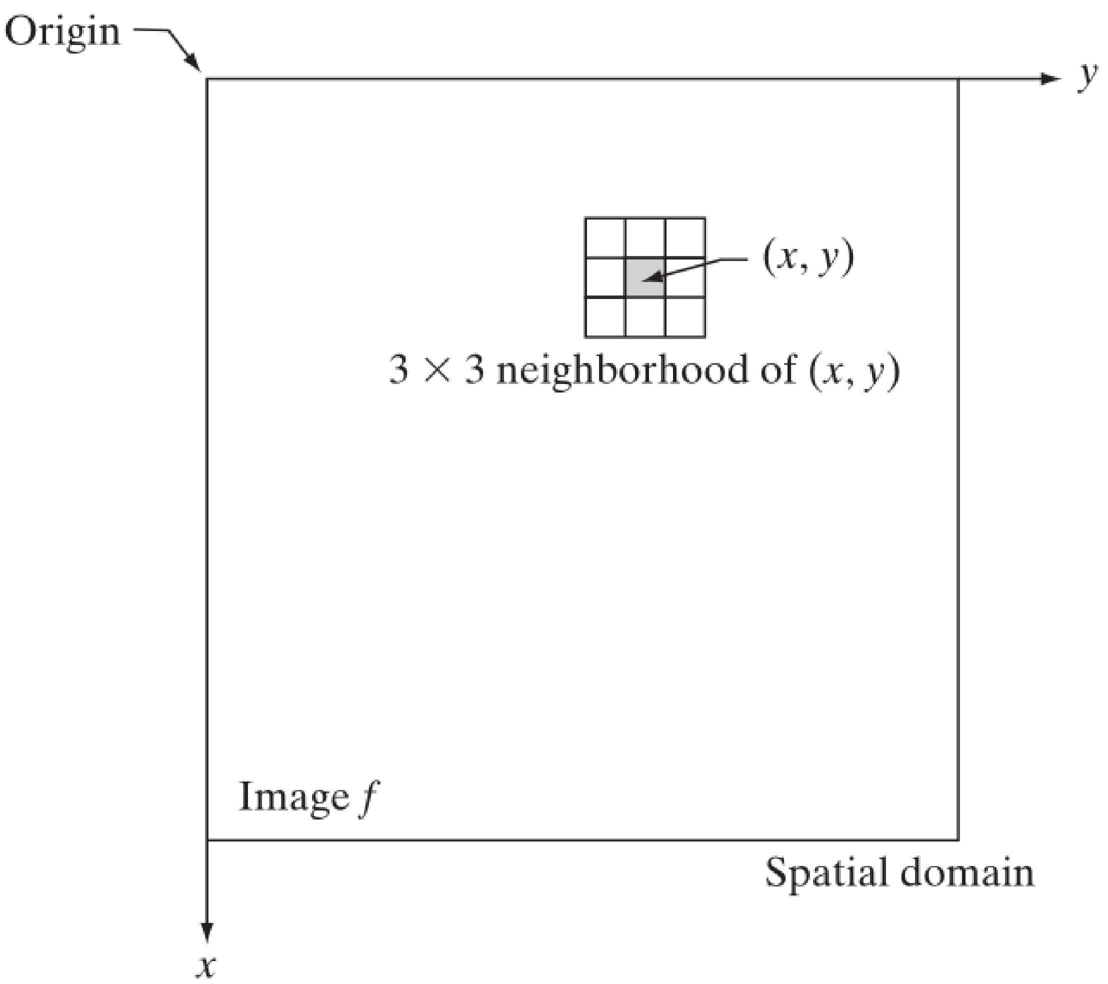
❑ The Mechanics of Spatial Filtering

➢ A filter consists of:

1) a neighborhood

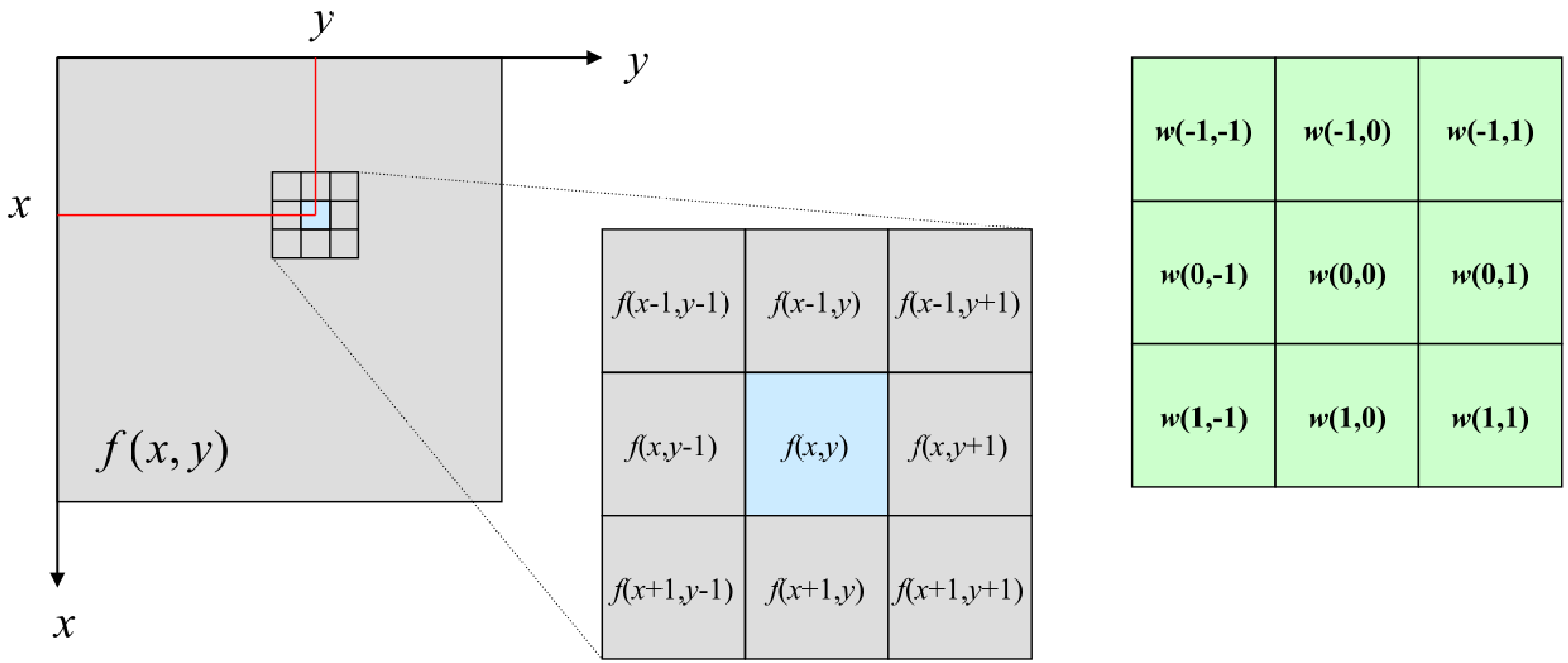
2) a predefined operation

- Linear spatial filter – if the operation performed on the image pixels is linear
- Nonlinear - otherwise



- Linear filtering with a filter mask

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)$$



❑ Spatial Correlation

$$g(x, y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s, t) f(x + s, y + t)$$

Step 1)

1	2	0	3	1
0	3	2	1	0
2	1	5	2	4
3	1	0	1	2
0	3	2	6	0

*

-1	-1	-1
0	0	0
1	1	1

⇒

5		

❑ Spatial Correlation

$$g(x,y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s,t) f(x+s, y+t)$$

Step 2)

1	2	0	3	1
0	3	2	1	0
2	1	5	2	4
3	1	0	1	2
0	3	2	6	0

*

-1	-1	-1
0	0	0
1	1	1

→

5	5	

❑ Spatial Correlation

$$g(x,y) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s,t) f(x+s, y+t)$$

Step 9)

1	2	0	3	1
0	3	2	1	0
2	1	5	2	4
3	1	0	1	2
0	3	2	6	0

*

-1	-1	-1
0	0	0
1	1	1

⇒

5	5	7
-1	-4	0
-3	3	-3

❑ Spatial Correlation

1	2	0	3	1
0	3	2	1	0
2	1	5	2	4
3	1	0	1	2
0	3	2	6	0

*

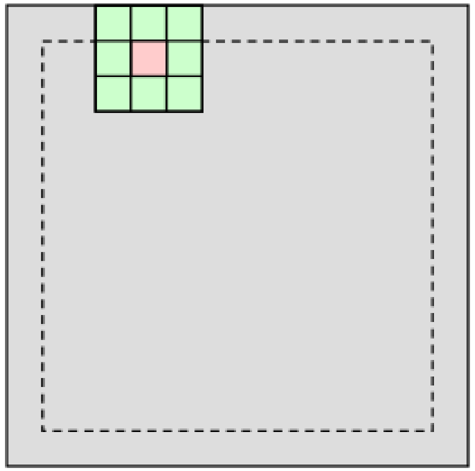
-1	-1	-1
0	0	0
1	1	1

⇒

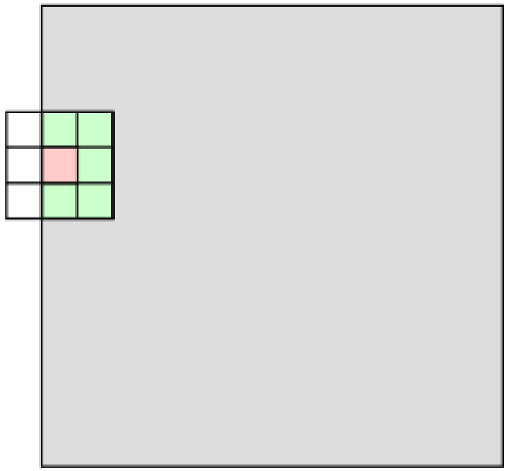
5	5	7
-1	-4	0
-3	3	-3

However, the image size changes

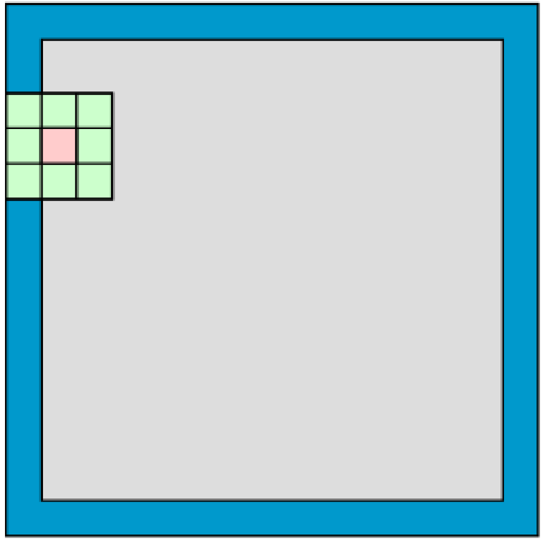
❑ Spatial Correlation



Limit Filtering



Partial Filtering



Padding 0 or constant

Replicating columns/rows

❑ Spatial Padding

1	1	2	0	3	1	1
1	1	2	0	3	1	1
0	0	3	2	1	0	0
2	2	1	5	2	4	4
3	3	1	0	1	2	2
0	0	3	2	6	0	0
0	0	3	2	6	0	0

*

-1	-1	-1
0	0	0
1	1	1

❑ Spatial Padding

1	1	2	0	3	1	1
1	1	2	0	3	1	1
0	0	3	2	1	0	0
2	2	1	5	2	4	4
3	3	1	0	1	2	2
0	0	3	2	6	0	0
0	0	3	2	6	0	0

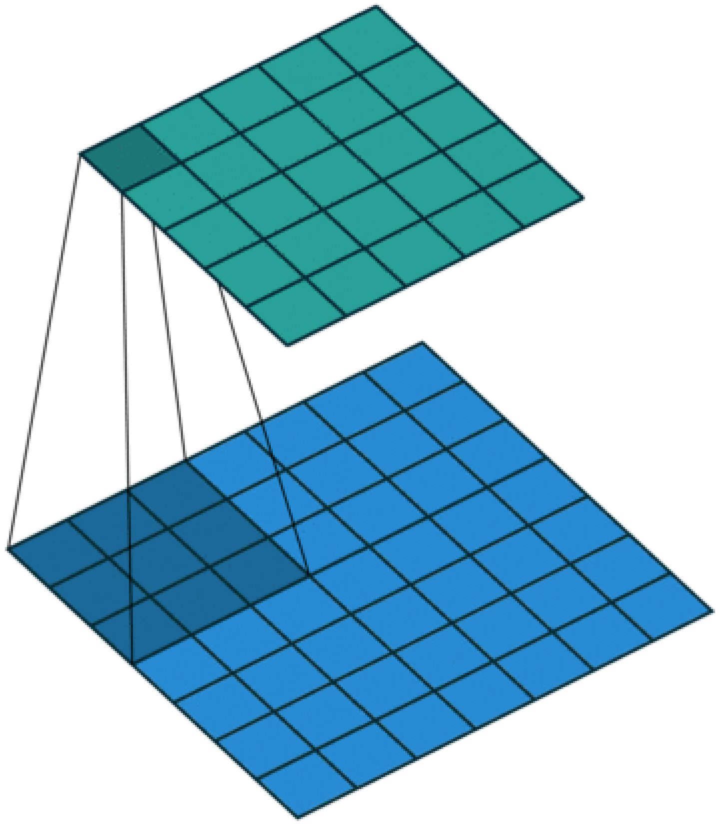
*

-1	-1	-1
0	0	0
1	1	1

⇒

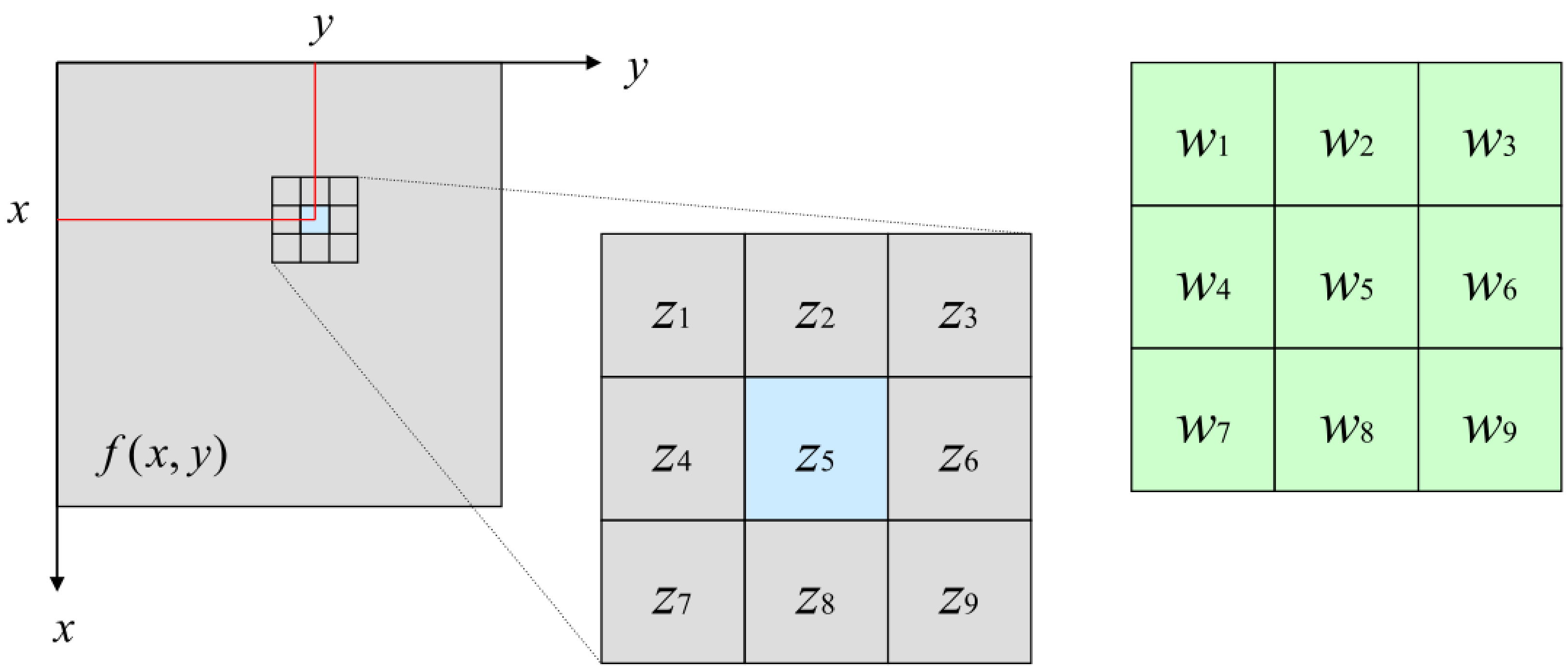
-1	2	1	-1	-4
1	5	5	7	5
4	-1	-4	0	4
-2	-3	3	-3	-4
-4	1	9	5	1

❑ Spatial Correlation



❑ Vector Representation of Linear Filtering

$$R = \sum_{i=1}^{mn} w_i z_i = \mathbf{w}^T \mathbf{z}$$



❑ Generating Spatial Filter Masks

- The filter coefficients are selected with different purposes
- e.g. averaging filtering

$$\frac{1}{9} * \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

- e.g. nonlinear filtering: max operation
  - a 5 * 5 max filter: the maximum intensity value of the 25 pixels and assigns that value to location (x, y)

Smoothing Spatial Filters

- Classification
  - Linear
  - Nonlinear

- Function of smoothing spatial filters
  - Blurring or Noise reduction

❑ Smoothing Linear Filters

- Output is the average of pixels in the neighborhood of the filter mask
- Function
  - ➢ Noise reduction
- Disadvantage
  - ➢ Undesirable side effect of blurring edges
- E.g.

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

- Smoothing Linear Filters

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)}$$

$\frac{1}{9} \times$

1	1	1
1	1	1
1	1	1

$\frac{1}{16} \times$

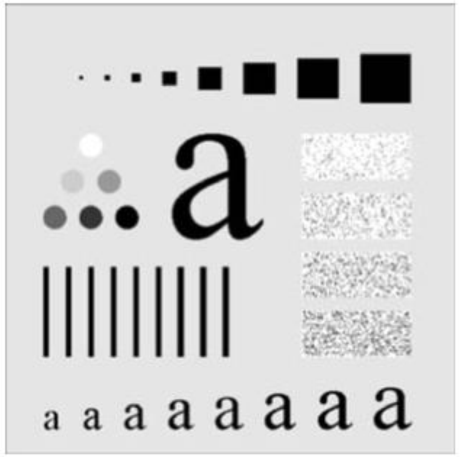
1	2	1
2	4	2
1	2	1

Weighted average

❑ Effect of Smoothing for Different Filter Size

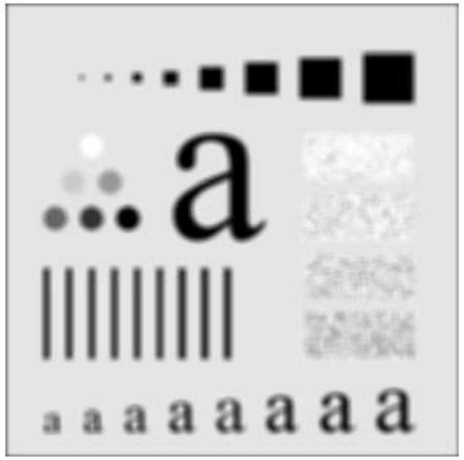
Original image

Filter size m=3

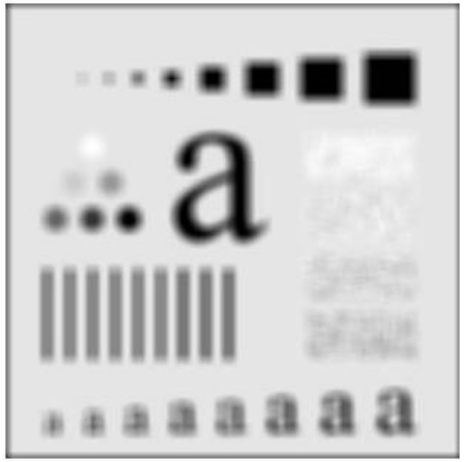


m=5

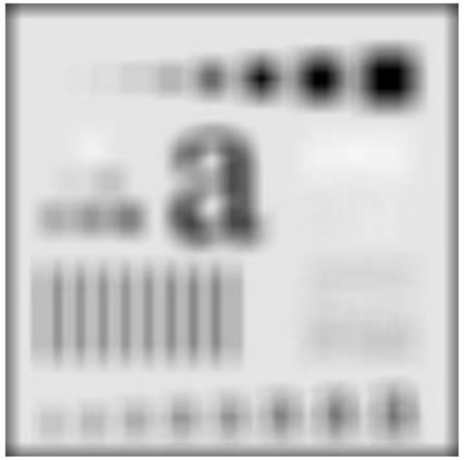
m=9



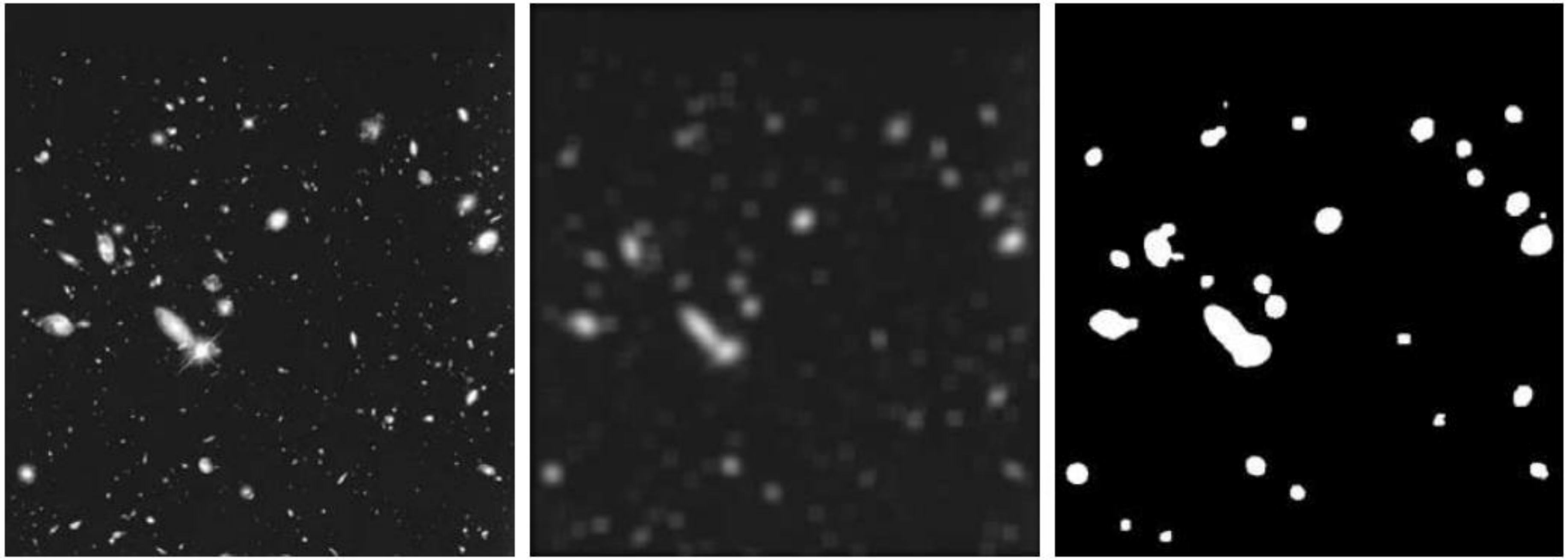
m=15



m=35



❑ Effect of Smoothing for Different Filter Size



Original image | 15 × 15 filter | After thresholding

Order-Statistic (Nonlinear) Filters

❑ Nonlinear filters

- Based on ordering (ranking) the pixels in the image area encompassed by the filter
- Commonly used
  - Median filter: effective for salt-and-pepper noise

10	20	20
20	100	15
20	20	25

Image area

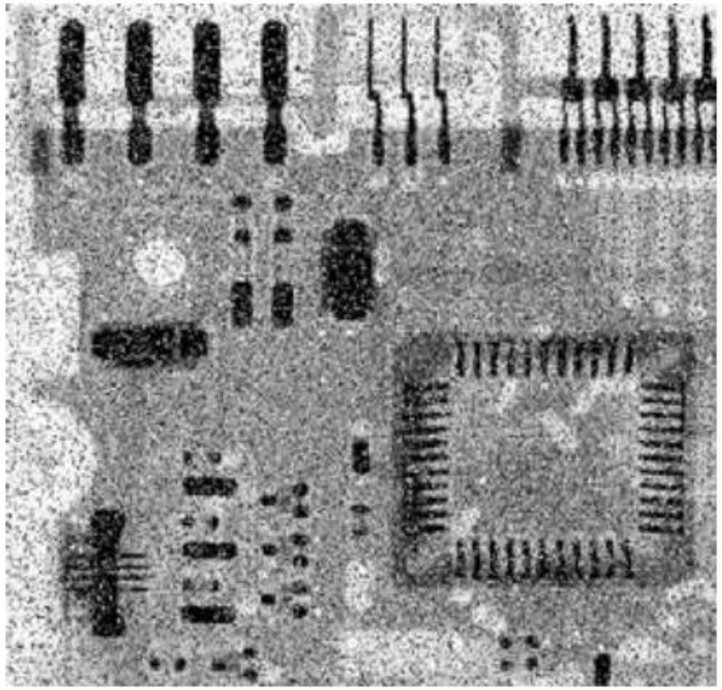
(10,20,20,20,100,15,20,20,25)

reordering

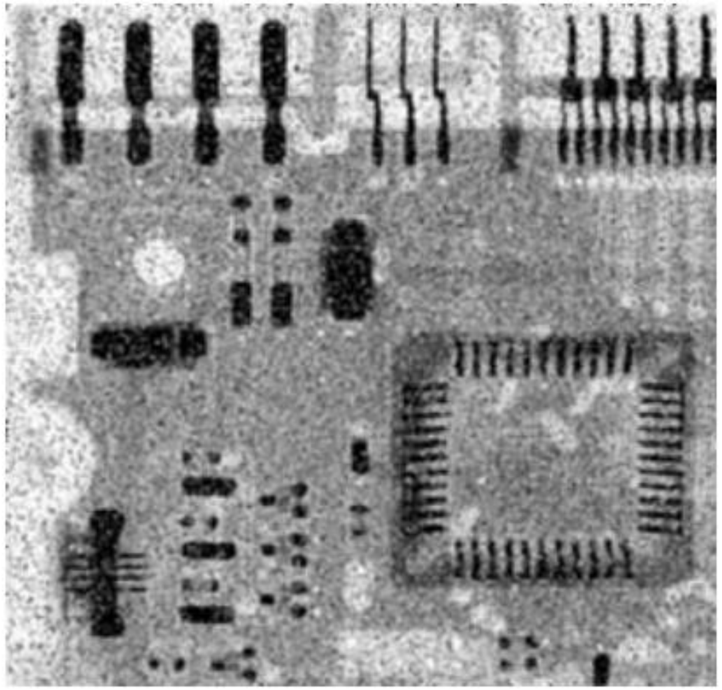
(10,15,20,20,**20**,20,20,25,100)

10	20	20
20	**20**	15
20	20	25

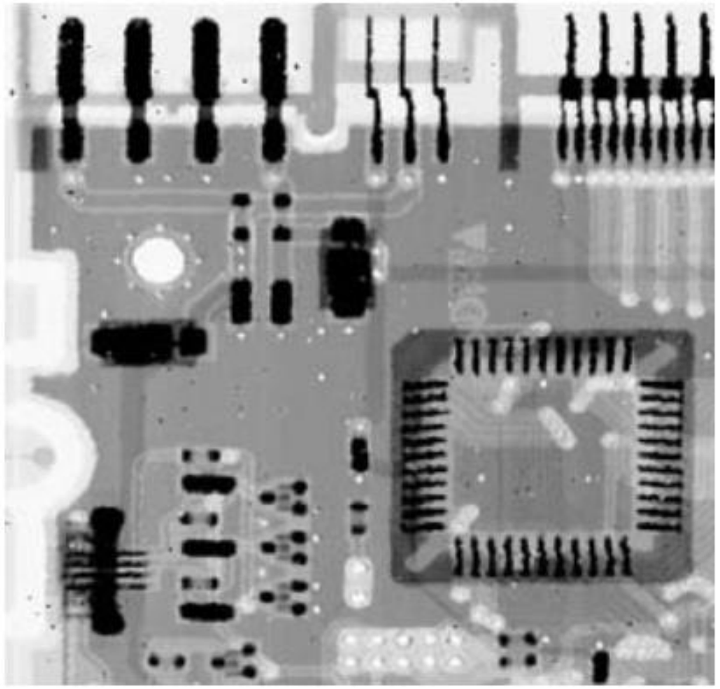
- Application



X-ray image with salt-and-pepper noise

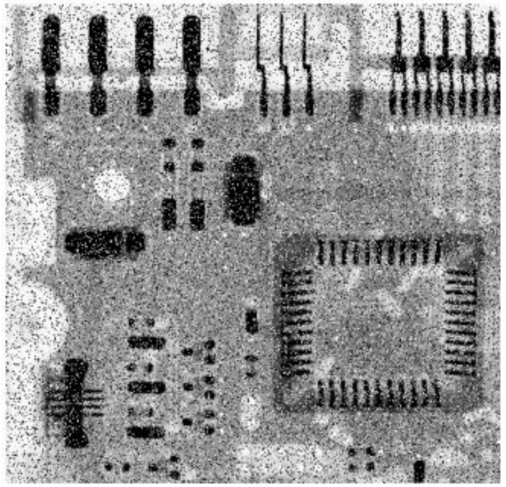


Average filtering of 3×3 size

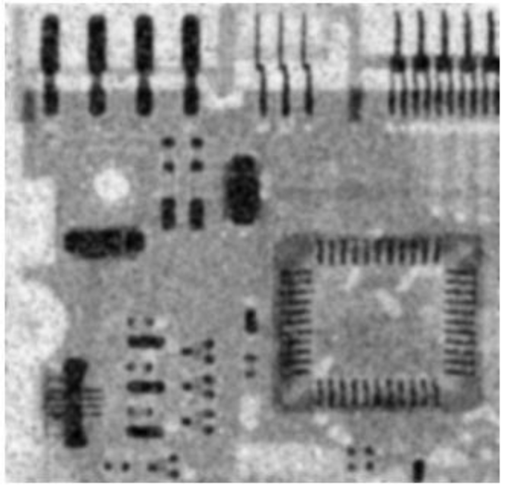


Median filtering of 3×3 size

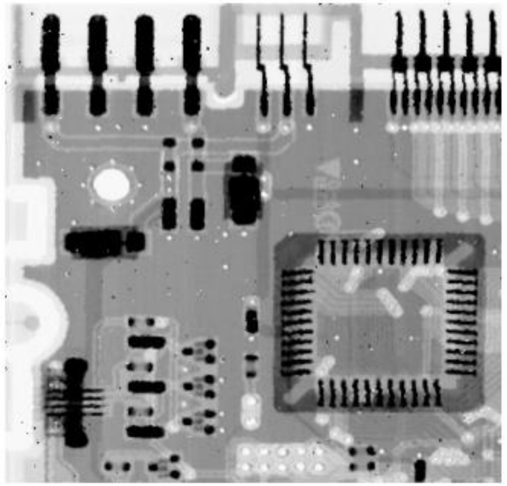
- Application



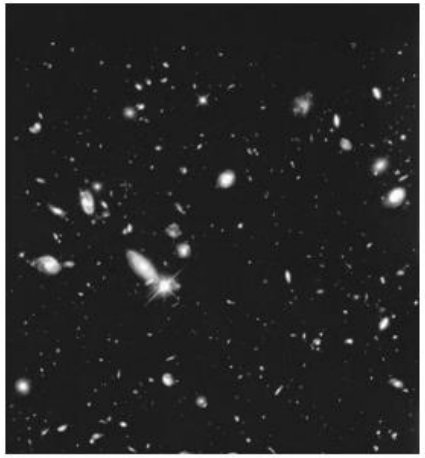
Salt-and-Pepper Noise



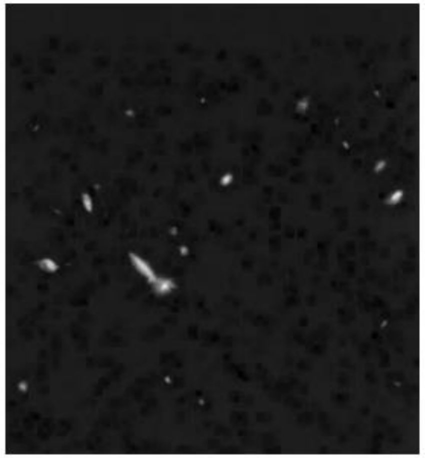
Gaussian Smoothing



Median Filtering



Original Image



Min Filtering

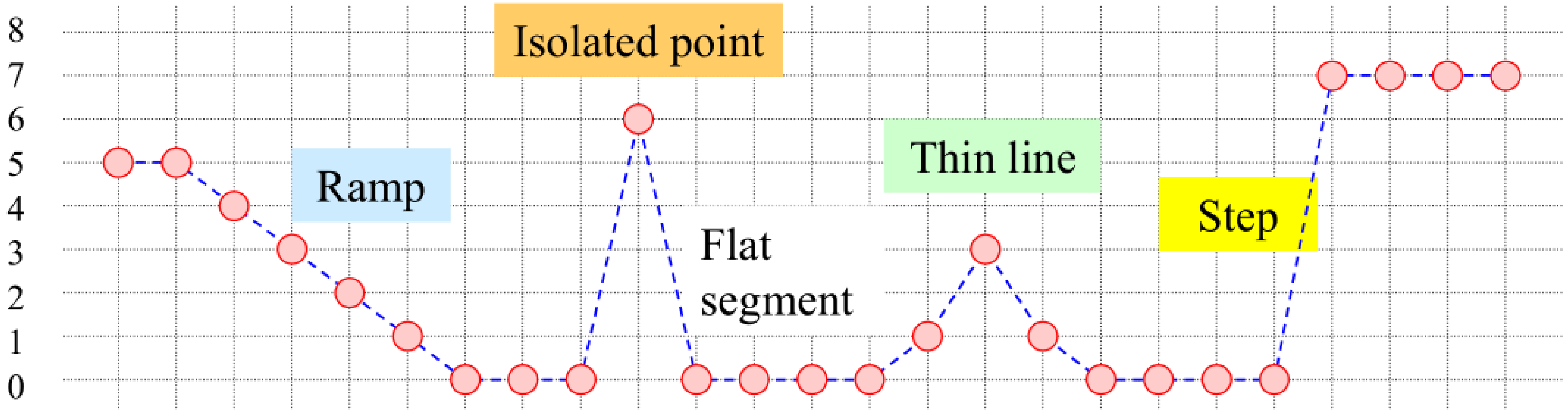
Min and Max Filtering

Sharpening Spatial Filters

- Sharpening
  - Highlight fine detail in an image or enhance detail that has been blurred
  - Spatial differentiation could be used for sharpening, while spatial integration for smoothing
- Effects of 1st and 2nd differences

$$\frac{df}{dx} = f(x+1) - f(x)$$

$$\frac{d^2 f}{dx^2} = f(x+1) + f(x-1) - 2f(x)$$



	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
1st diff		-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	
2nd diff		-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	

Thick Edges

Stronger Response to fine detail

0	3	7	3	0
3	4	-4	-3	0
3	1	-8	1	3

Double Edges

❑ Second Derivative – Laplacian

- Isotropic: rotation invariant

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Linear operator

$$\nabla^2 (af(x,y) + bg(x,y)) = \nabla^2 (af(x,y)) + \nabla^2 (bg(x,y)) = a\nabla^2 f + b\nabla^2 g$$

- Discrete form

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Sharpening Spatial Filters

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= f(x+1,y) + f(x-1,y) +$$

$$f(x,y+1) + f(x,y-1) - 4f(x,y)$$

0	1	0
1	-4	1
0	1	0

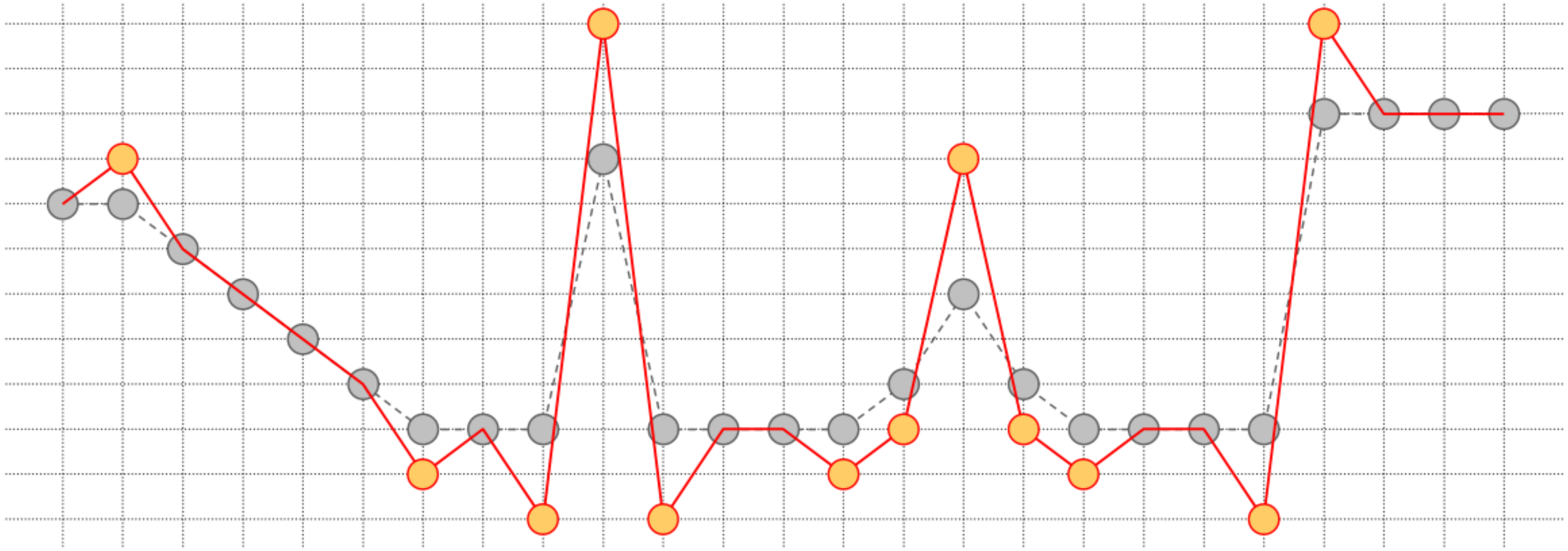
1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

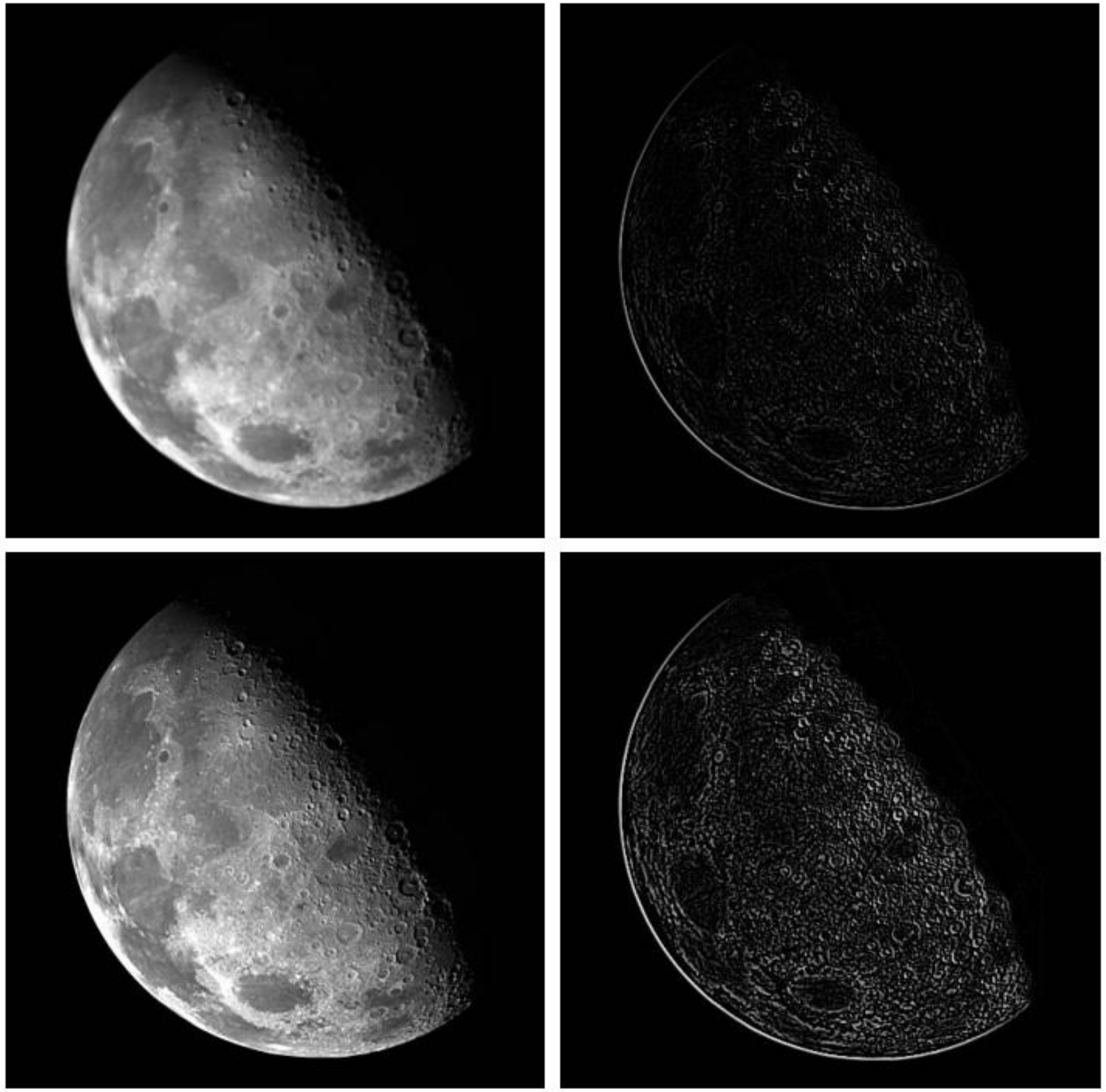
❑ Sharpening with the Laplacian

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if negative center} \\ f(x,y) + \nabla^2 f(x,y) & \text{if positive center} \end{cases}$$



	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
2^{nd} diff		-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	

Sharpening Spatial Filters



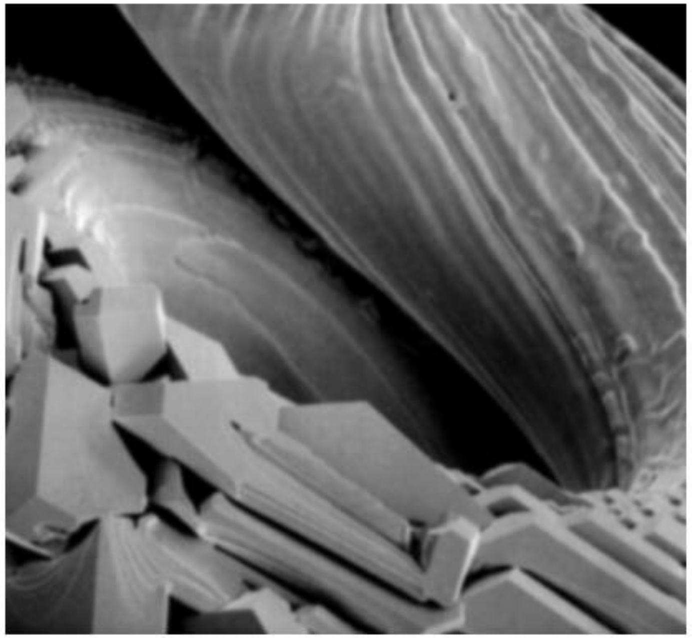
- Simplification using a mask

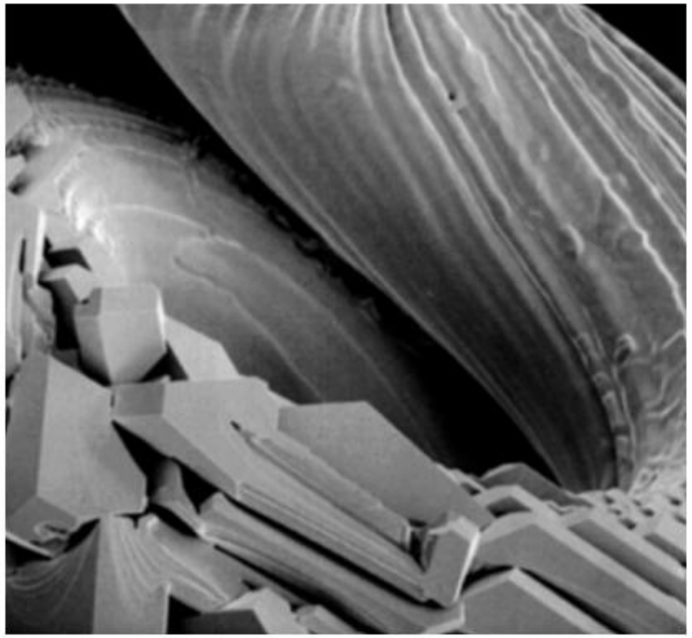
$$
\begin{aligned}
&g(x,y) \\
&= f(x,y) + \nabla^2 f \\
&= 5f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]
\end{aligned}
$$

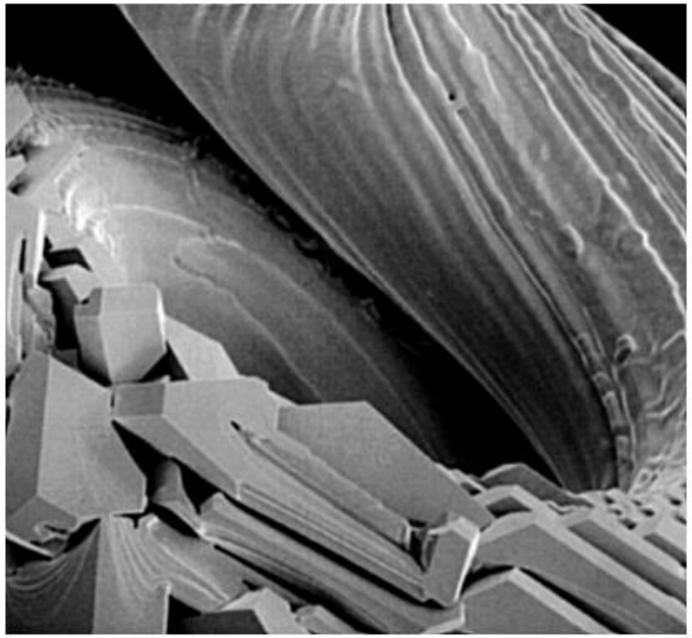
0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

Sharpening Spatial Filters





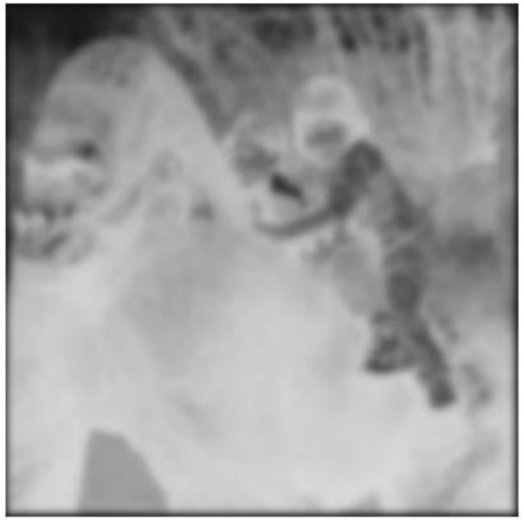


0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

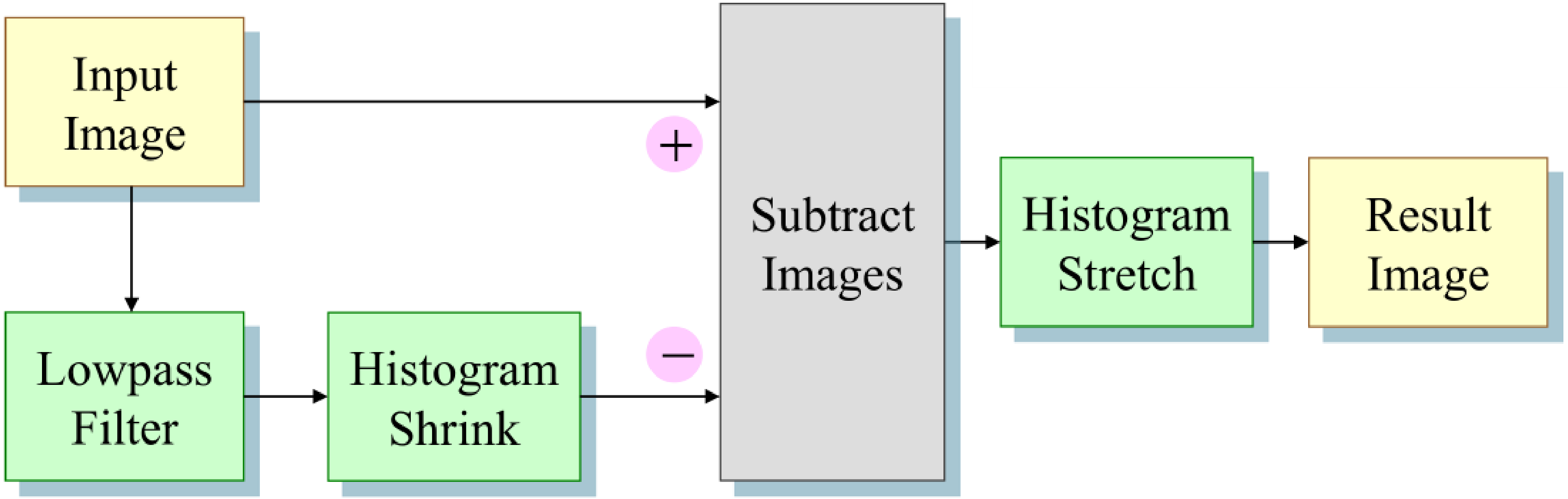
❑ Unsharp Masking

- It sharpens the image by subtracting a blurred (lowpass) version of the original image
- Photographers used it for many years to enhance images
- It was accomplished during film development by superposing a blurred negative onto the corresponding positive film to produce a sharper result

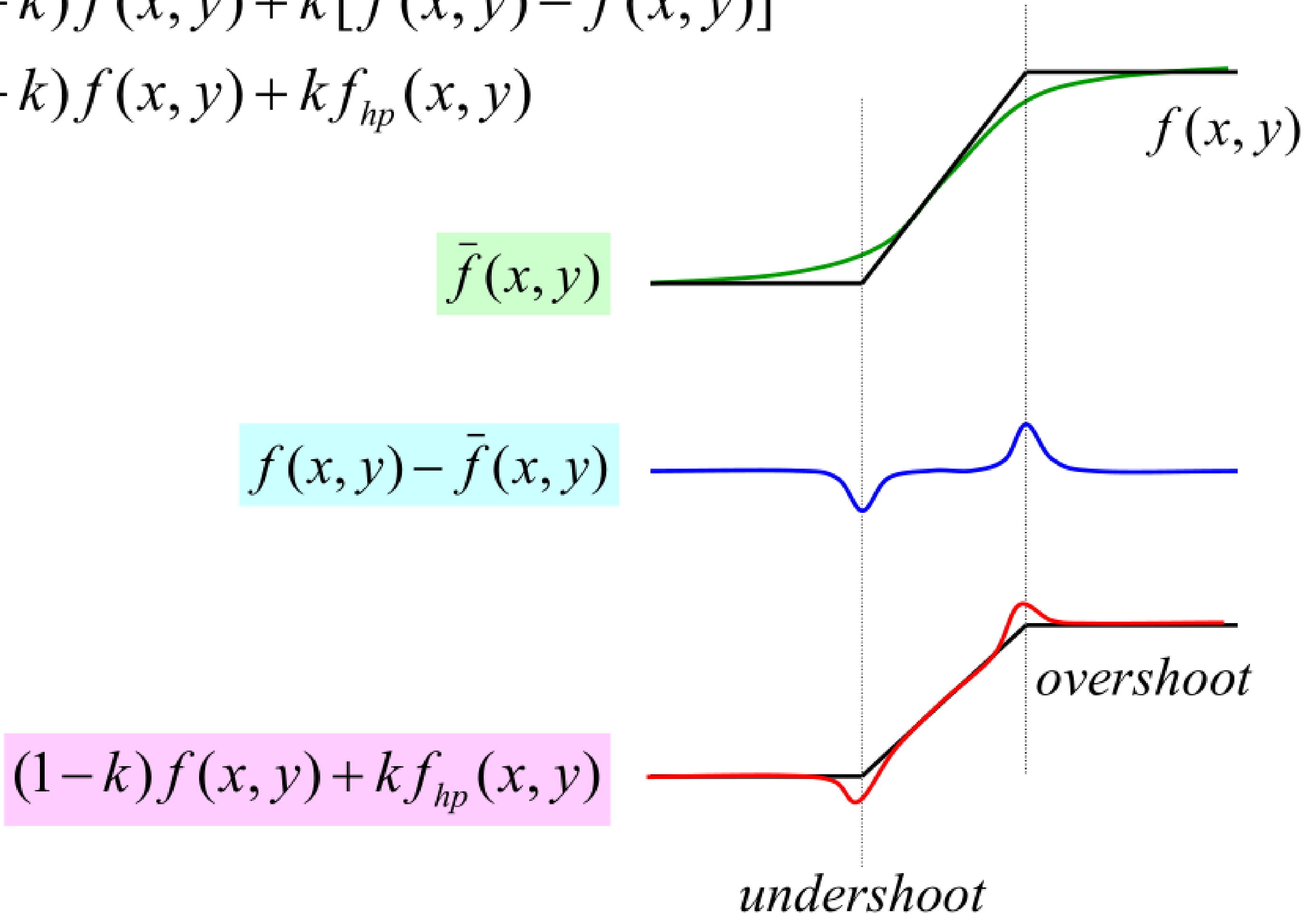




- The process is similar to adding a detail enhanced (highpass) version of the image to the original
- This process has visual effect of causing overshoot and undershoot at the edges, which has the effect of emphasizing the edges

$$f_{usm}(x,y) = f(x,y) - k\bar{f}(x,y)$$

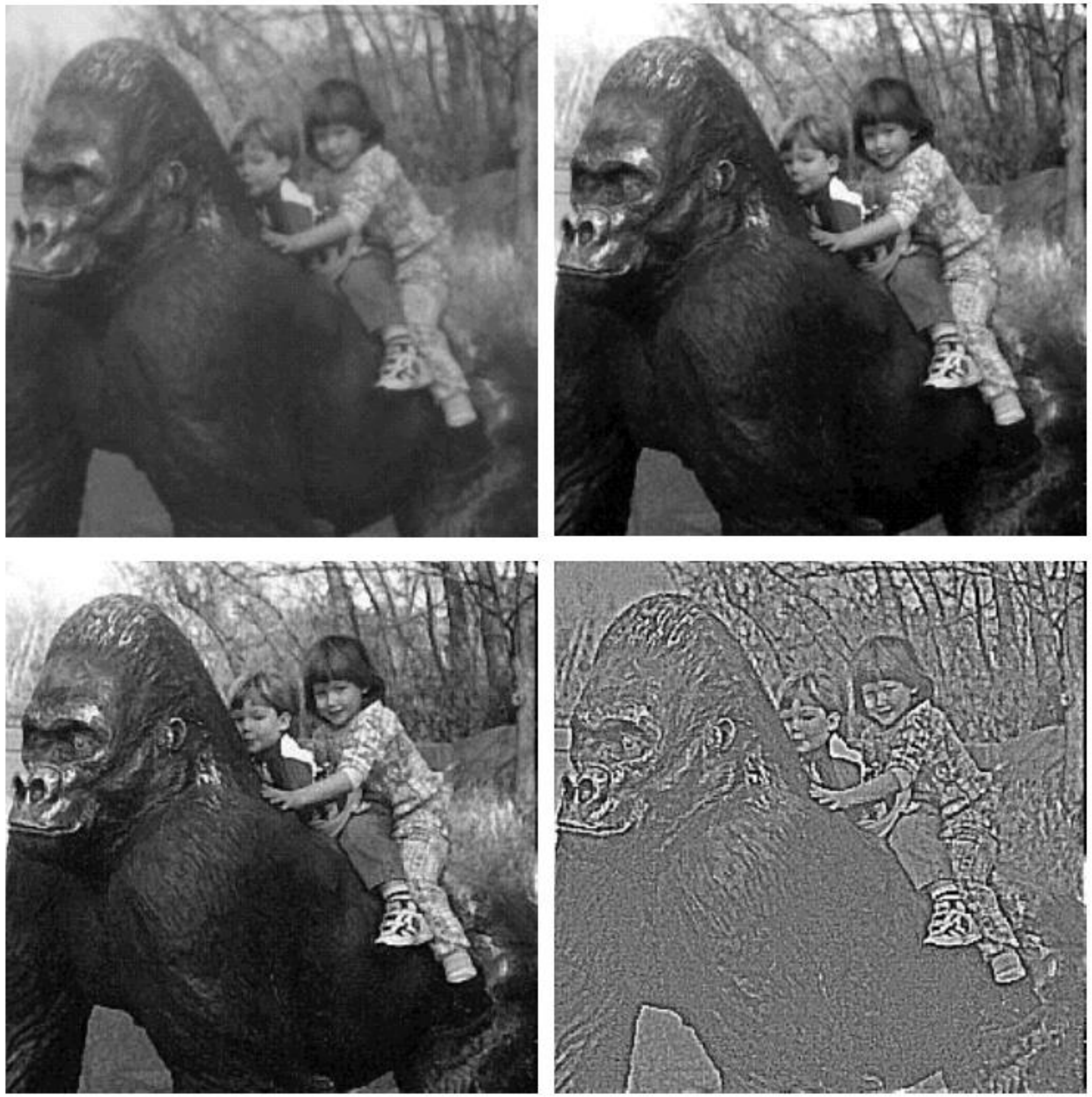


$$f_{usm}(x,y) = f(x,y) - k\bar{f}(x,y)$$
$$= (1-k)f(x,y) + k[f(x,y) - \bar{f}(x,y)]$$
$$= (1-k)f(x,y) + kf_{hp}(x,y)$$



Sharpening Spatial Filters

Original Image



Unsharp masking
lower limit=0, upper=100, 2% low and high clipping

$k \approx 0.4$

Unsharp masking
lower limit=0, upper=150, 2% low and high clipping

$k \approx 0.6$

Unsharp masking
lower limit=0, upper=200, 2% low and high clipping

$k \approx 0.8$

❑ High-Boost Filtering

$$f_{hb}(x,y) = Af(x,y) - \bar{f}(x,y)$$
$$= (A-1)f(x,y) + f(x,y) - \bar{f}(x,y)$$
$$= (A-1)f(x,y) + f_{hp}(x,y)$$

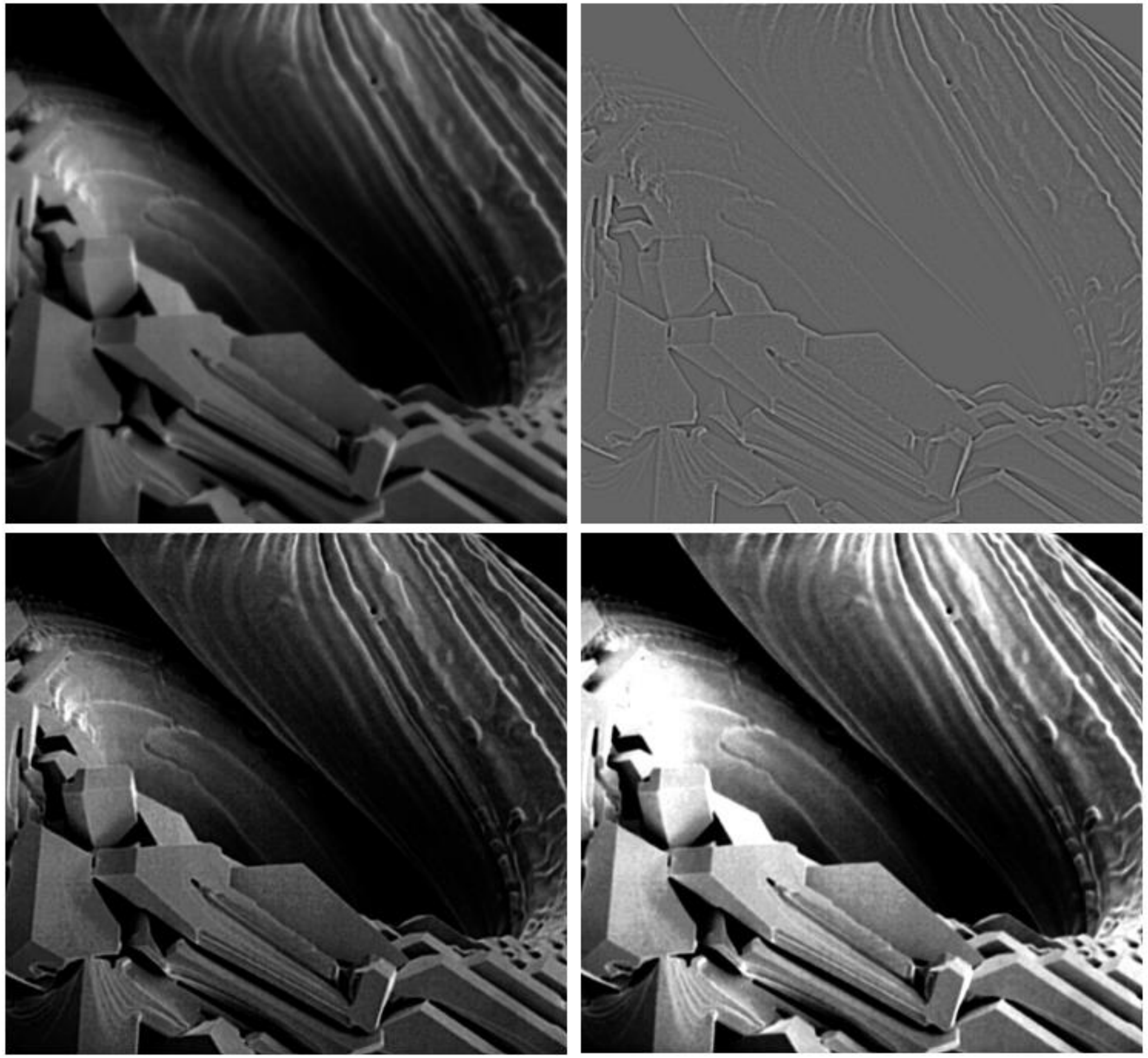
$$f_{hb}(x,y) = \begin{cases} Af(x,y) - \nabla^2 f(x,y) & \text{if negative center} \\ Af(x,y) + \nabla^2 f(x,y) & \text{if positive center} \end{cases}$$

0	-1	0
-1	A+4	-1
0	-1	0

-1	-1	-1
-1	A+8	-1
-1	-1	-1

Sharpening Spatial Filters

0	-1	0
-1	A+4	-1
0	-1	0



$A = 0$

$A = 1$

$A = 2$

❑ First-Order Derivative (Gradient)

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\begin{aligned} \|\nabla f\| &= mag(\nabla f) = \left(G_x^2 + G_y^2\right)^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2} \\ &\approx |G_x| + |G_y| \end{aligned}$$

- Discrete form

Roberts Operator

-1	0
0	1

0	-1
1	0

Prewitt Operator

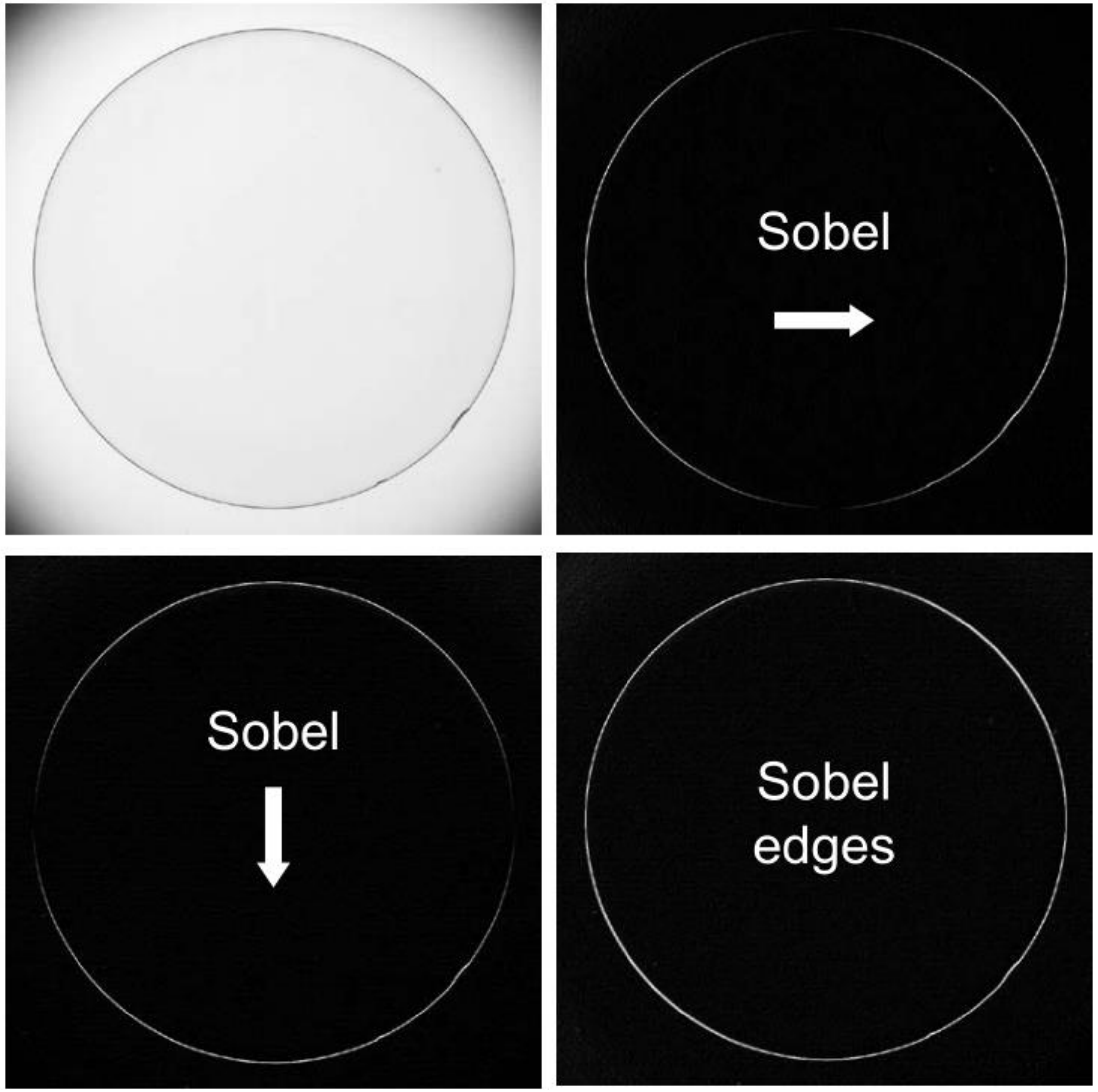
-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Sobel Operator

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1


Sobel
Sobel
Sobel
edges

Summary

- Spatial Filtering
  - Fundamentals of Spatial Filtering
  - Smoothing Spatial Filters
  - Sharpening Spatial Filters

- Next: Filtering in the Frequency Domain

Q & A

Thank You!